

# The need for geological and petrophysical constraints in geophysical inversion

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# Acknowledgements

Prominent Hill inversion results presented by kind permission of OZ Minerals

Spectrem example courtesy Anglo American Exploration

Mt Dore study courtesy Geological Survey of Queensland

Bull Creek data provided by Exco Resources



# Outline

- Introduction
- Constrained inversion as a geological modelling tool
- Geological and petrophysical constraints
- Opportunities and roadblocks
- Conclusions

# Introduction

- Unconstrained inversion
- Constrained inversion
- Stochastic inversion

# “Unconstrained” inversion

We define “unconstrained” inversion as a computational process used to find one model which satisfies a geophysical data set alone.

The starting model is homogeneous.

In fact “unconstrained” inversion is always constrained implicitly, e.g. by model cell sizes, default conditioning (depth penalty function, smoothness)

“Unconstrained” inversion may also be constrained explicitly, e.g. via property bounds (including positivity)

## “Unconstrained” inversion cont’d

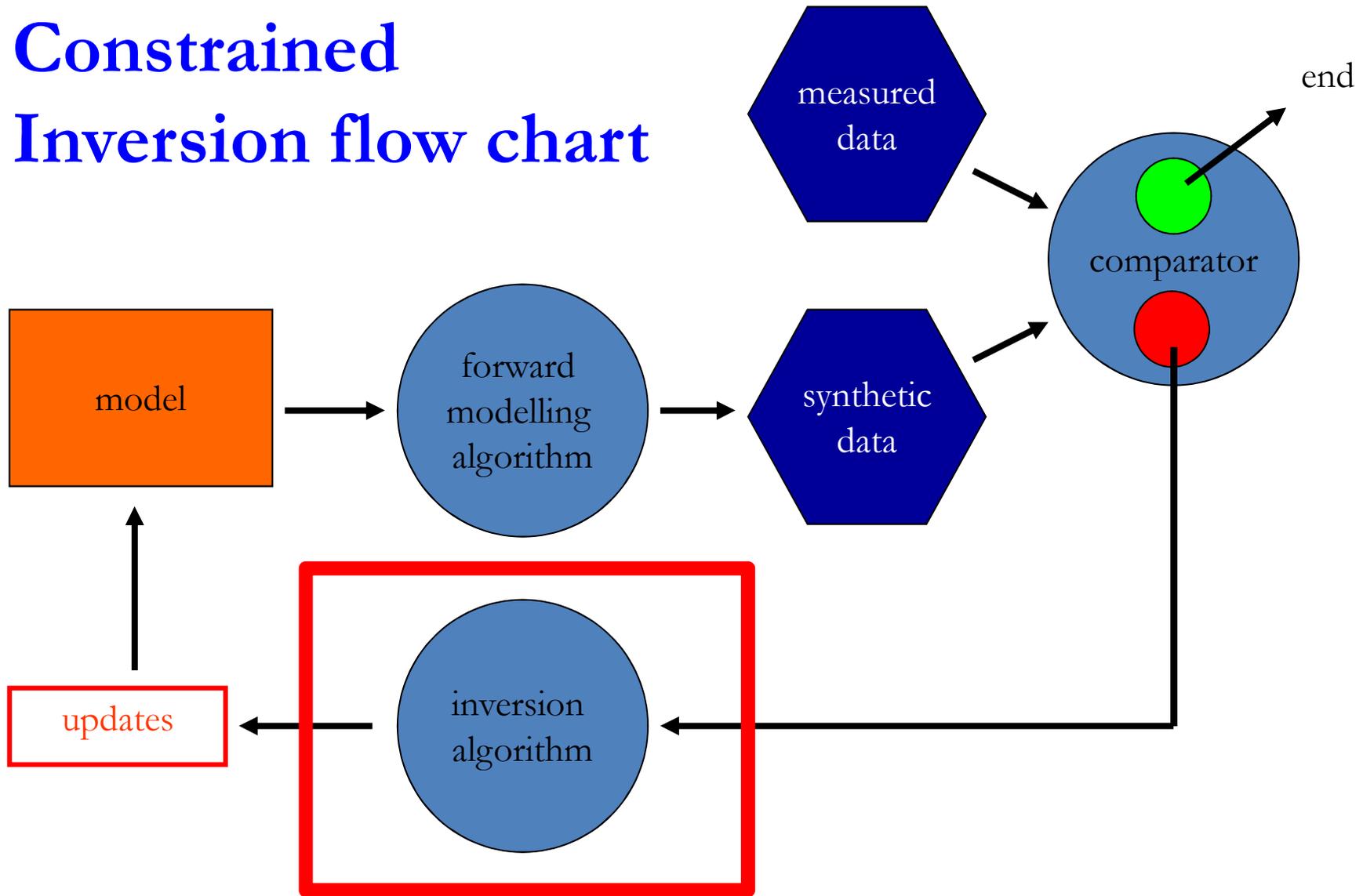
“Unconstrained” inversion of geophysical data in isolation may be effective for targeting.

Suitable for production line: data QC and first-pass interpretation

Deterministic

Typical users: contractors, mining companies

# Constrained Inversion flow chart



# Constrained inversion

We define “constrained” inversion here as a interpretational process used to find a model which satisfies the geophysical data while honouring all available geological information.

The starting model is geologically-inspired.

Constraints may be “hard”, enforcing observations, or “soft, favouring characteristics

Typical users: consultants, mining companies

# Constrained inversion cont'd

According to our definition, constrained inversion is deterministic, in keeping with conventional geological modelling

Exploration decisions usually based on a single “best information” geological model: deterministic

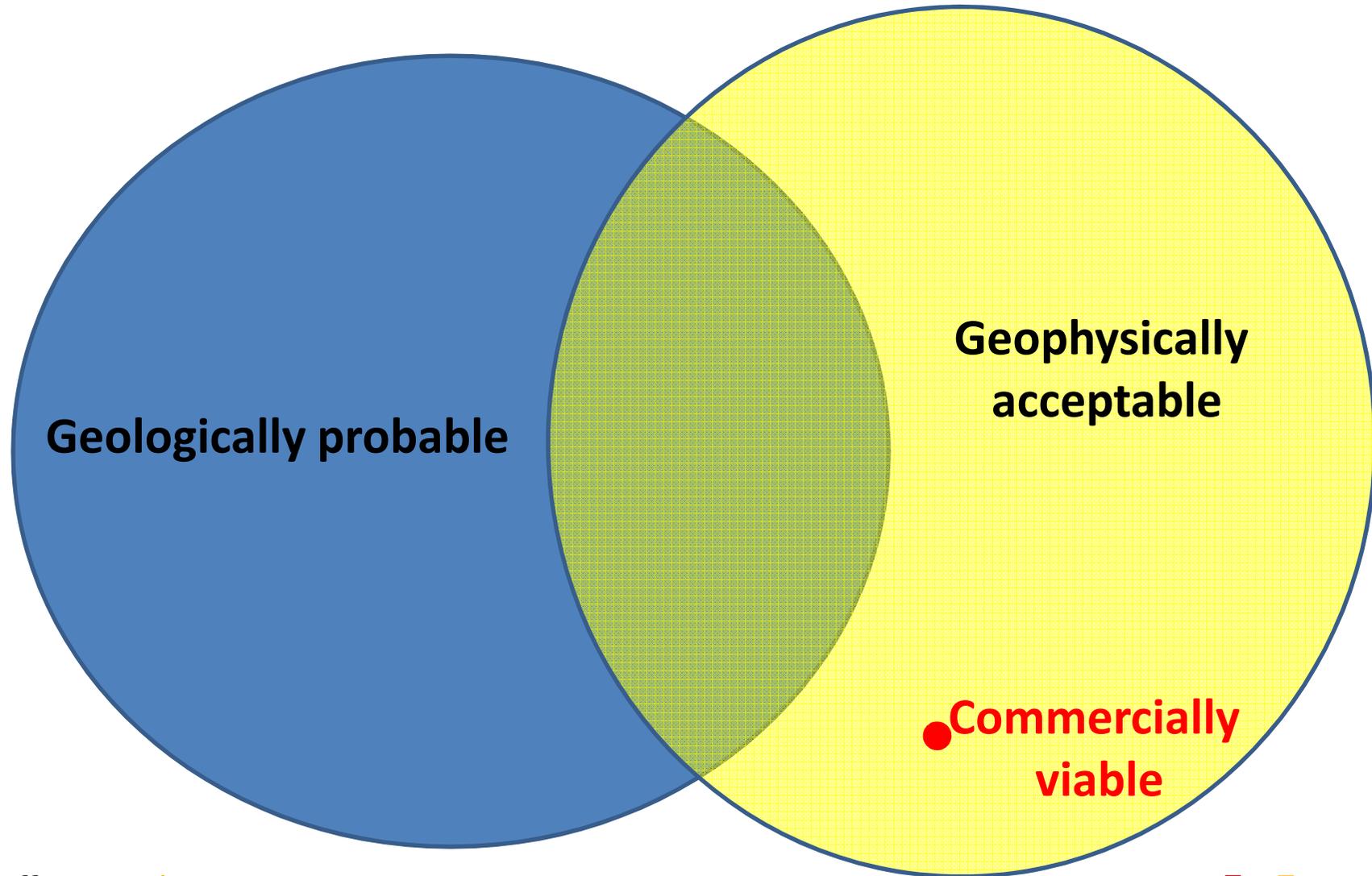
Stochastic inversion can be constrained also.

“Constrained deterministic” is unwieldy

# Stochastic inversion

- Stochastic approaches endeavour to characterise the entire suite of possible models in statistical fashion
- Probabilistic approach is computationally onerous and probabilities themselves are uncertain.
- Typical users: universities, government agencies
- Deterministic methodologies carry additional risk, but exploration driven by possibility rather than probability
- The most probable model may not be very interesting

# The elusive exploration target



## Part 1

# Constrained inversion as a geological modelling tool

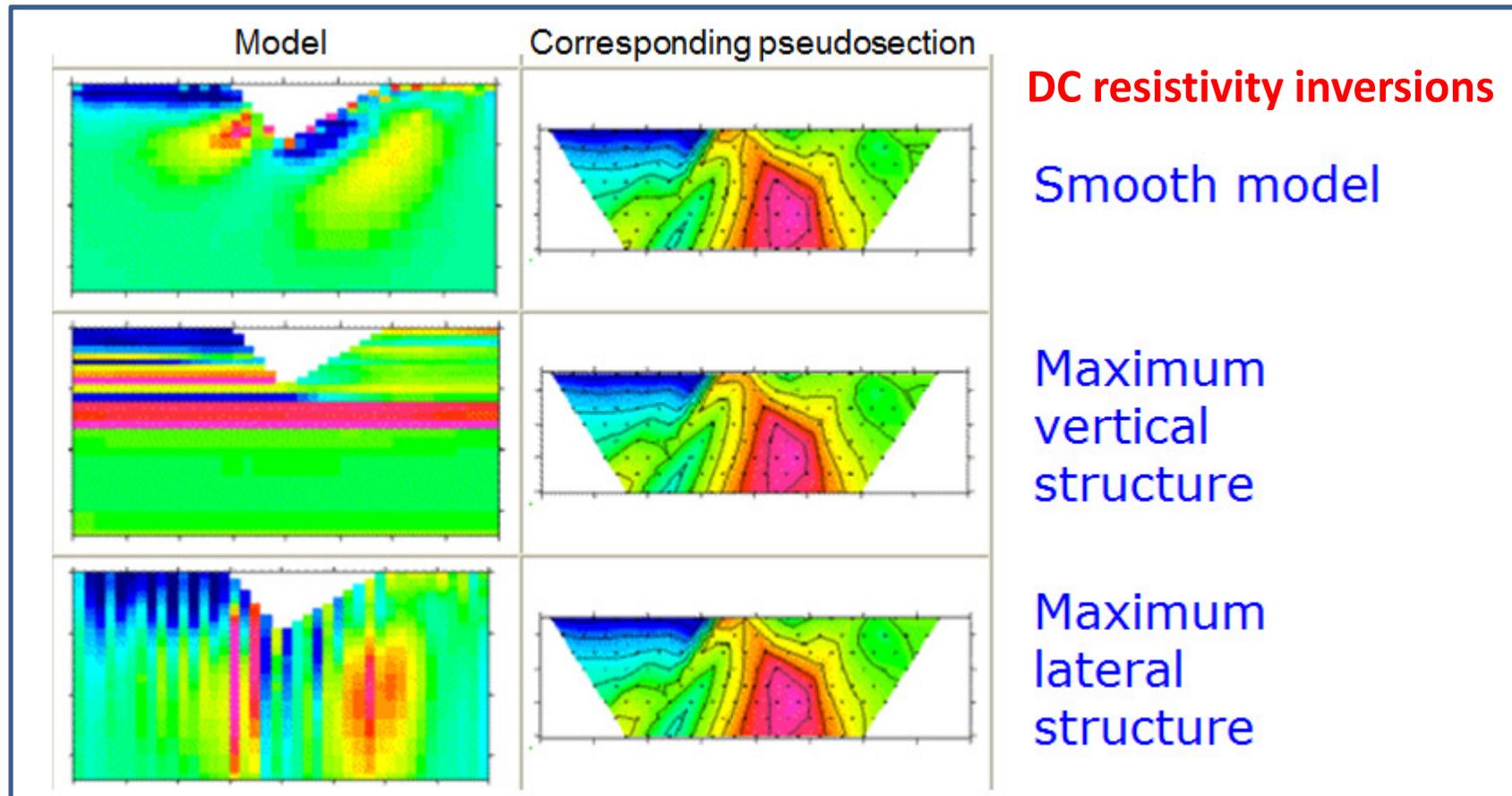
- Inversion as a component of Earth modelling
- Advantages of inverting a geological model

# Inversion

## as a component of Earth modelling

- Geophysics is increasingly deployed as a mapping tool, to deliver an improved geological “map” of the area of interest.
- The role of inversion is to extend the map into the 3<sup>rd</sup> dimension ... hence to create or refine a 3D geological model
- Inversion can be regarded as a component in an overarching effort to build, ideally, a cross-disciplinary interpretation, aka “common Earth model” (CEM)

# Non-uniqueness

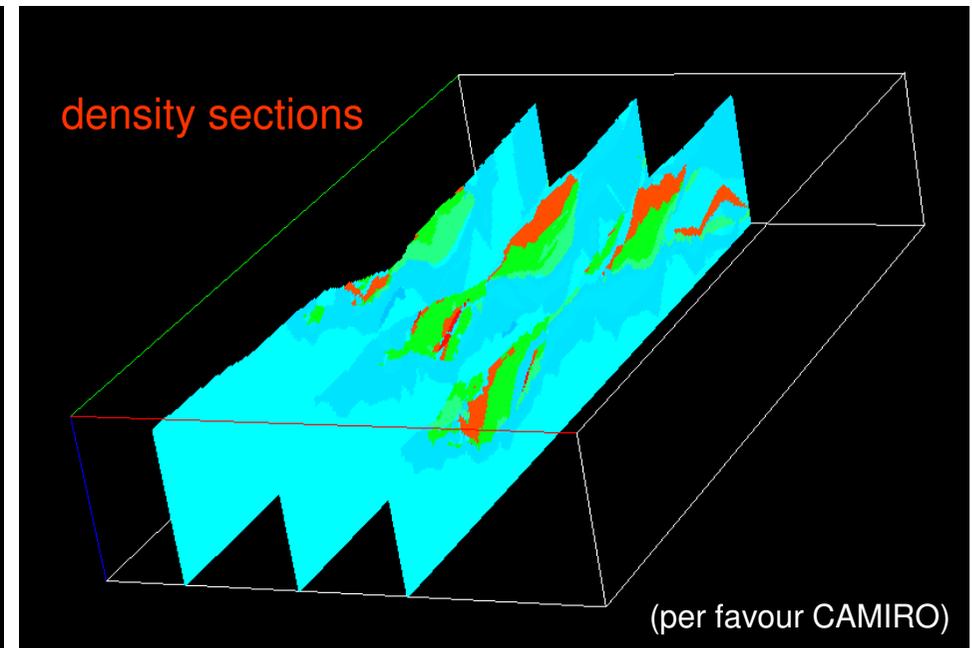
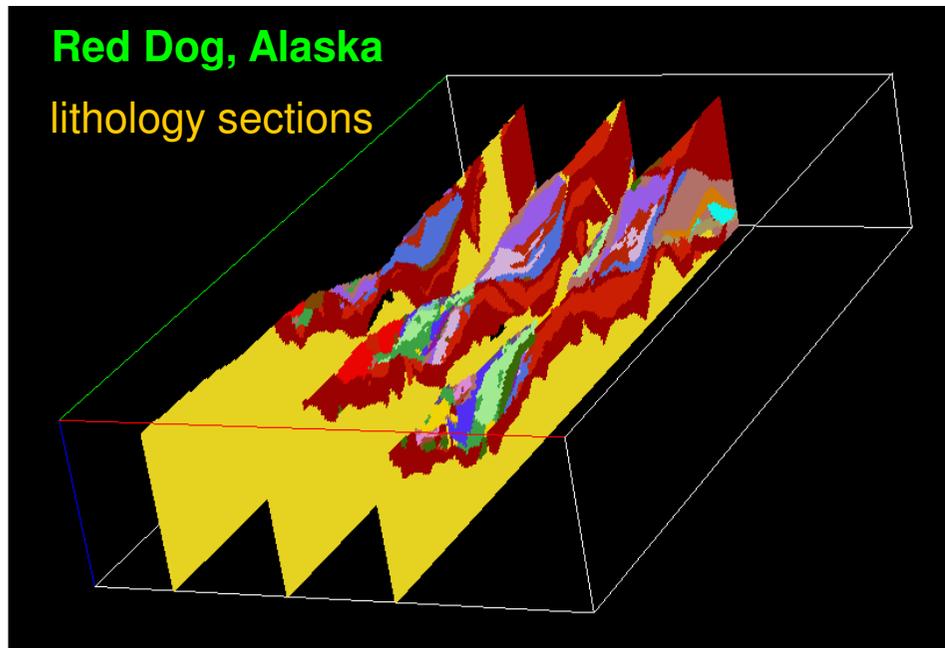


# Non-uniqueness

Ambiguity not a purely geophysical phenomenon

## Sources of non-uniqueness:

1. Intrinsic physical limitations
2. Finite extent and spatial density of data
3. Errors on the data
4. [Lack of property contrast: S/N, geological ambiguity]



# Uniqueness & the role of constraints

Geophysics maps variations in rock properties.

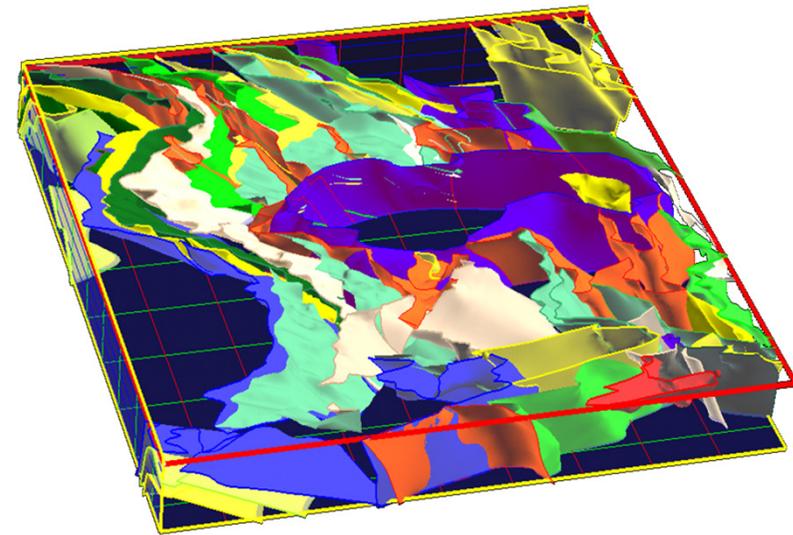
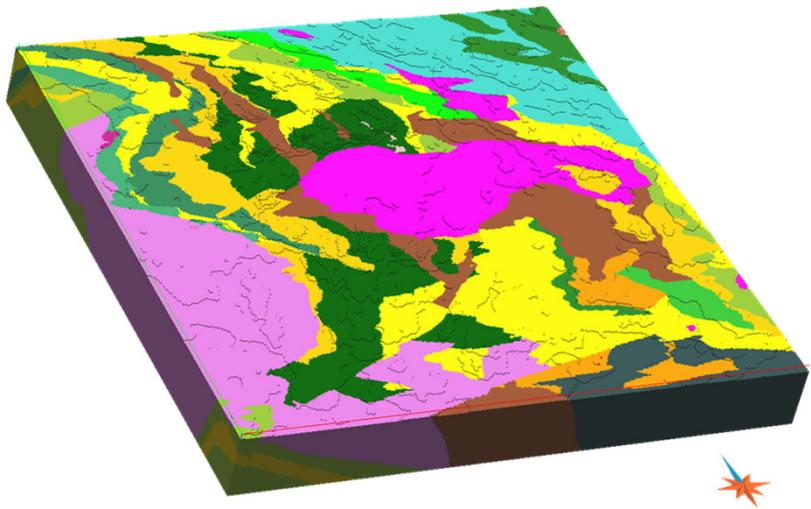
There are usually an infinite number of rock property models satisfying the geophysical data acceptably well.

Constraints required in order to

1. incorporate what is known about the geology, and
2. reject models inconsistent with what is already known.

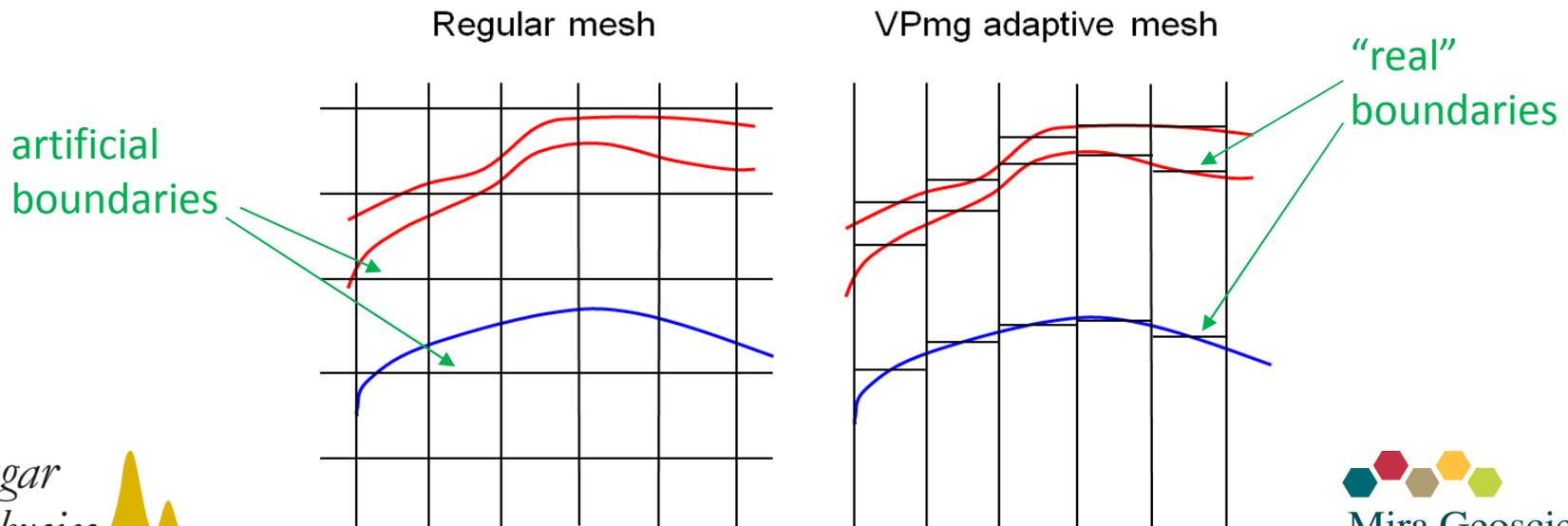
# Geological models

- Geological models are comprised of rock type domains, their attributes, and the boundaries (contacts, structures) which enclose them
- Geological models are categorical, insofar as each domain is assigned to a rock type



# Geological models

- Models for geophysical inversion must be petrophysical, and can also be geological
- In conventional petrophysical (or property) models the subsurface is divided into cells with one or more physical properties, but no rock type.
- Cell boundaries in petrophysical models are often artificial, i.e. bear no relation to geological contacts and structures.

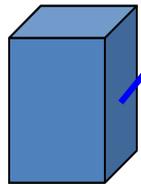
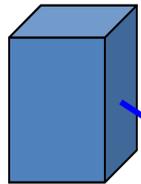
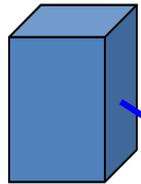
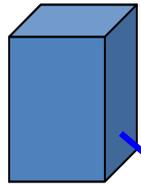


# Geological modelling

- Commonly a recursive process in response to successive campaigns of data acquisition – geological (drilling), geochemical, geophysical
- The first stage is usually surface-building
- Inversion too can be viewed as a sequence of recursive steps – not a single program run
- If applied to a geological model, inversion becomes one of the modelling options in the toolbox

# Geological model for inversion

Model cells

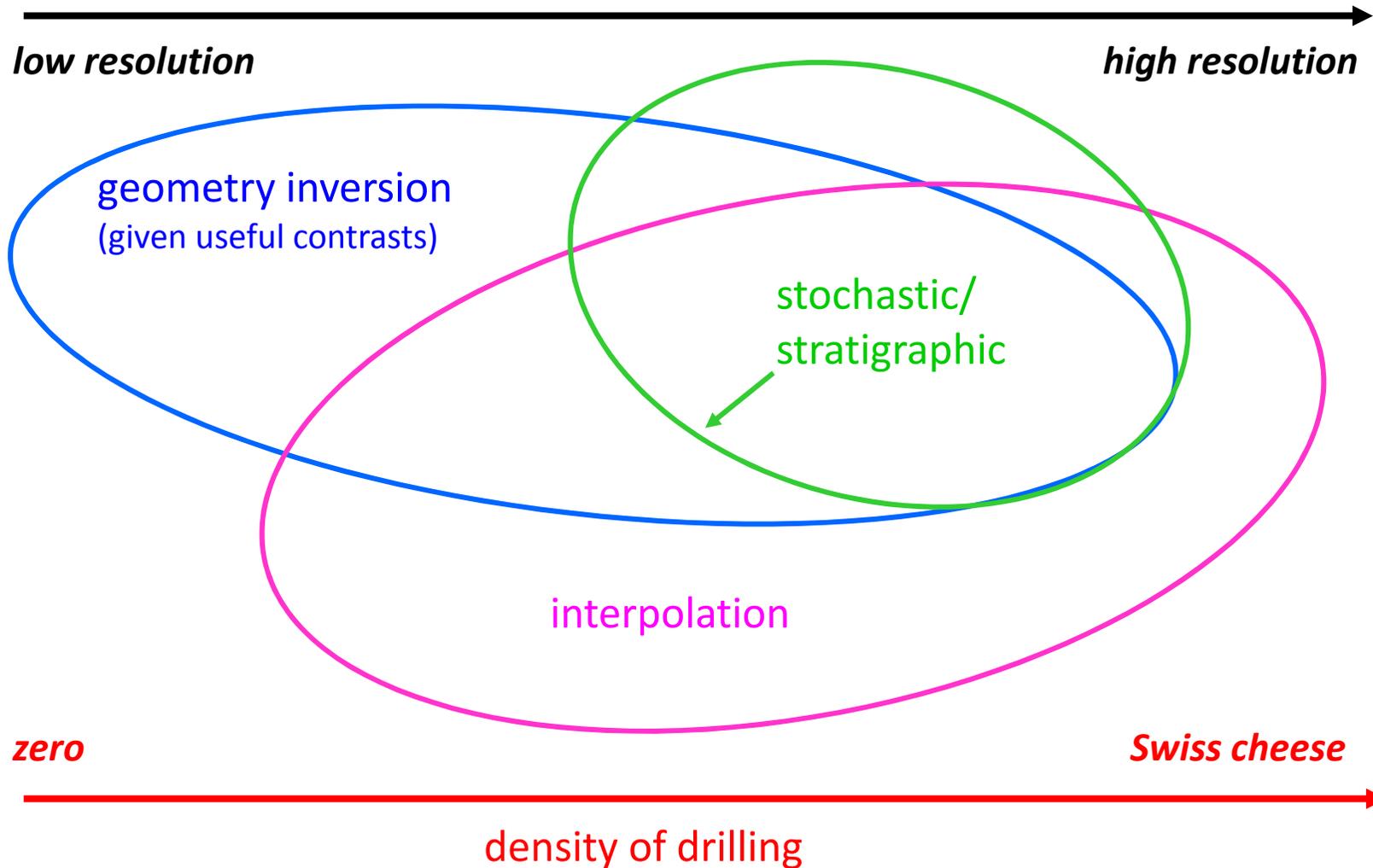


Cells have rock type as well as property

rock unit	mean	min	max	dec	inc	Q	pdf
Severne sandstone	1	0	50	0	0	0	
Cambrian	2	0	20	0	0	0	
felsic volcanic	5	1	10	0	0	0	
dolerite	50	5	100	25	-69	1.2	
granite	3	1	25	0	0	0	
fault zone	1	0	30	0	0	0	
breccia	3	0	10	0	0	0	
massive mt	3500	2700	4000	40	30	1	
disseminated cpy	0	0	5	0	0	0	

Categorical VPmg model structure

# Surface Modelling Options



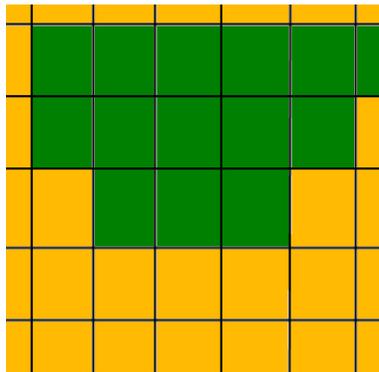


# Advantages of inverting a geological model

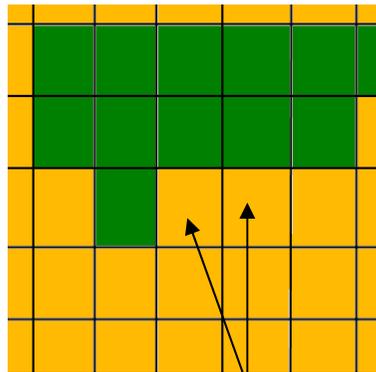
- Strong driver for integration
- Natural constraint assignment
- **Geometry** inversion as well as **property** inversion
- Fast optimisation of homogeneous properties
- Greater control of inversion volume
- Assign magnetic remanence to geological units
- Assign statistical distributions according to rock type
- Regard as either geology or property model
- “Unconstrained” inversion still an option

# Geometry inversion

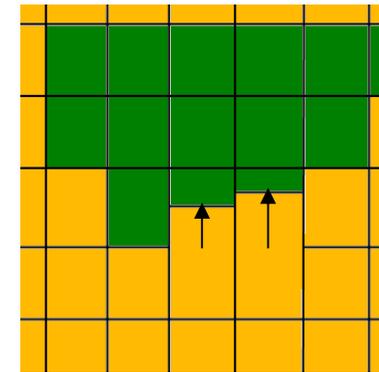
- **Generally achieved either by...**
  - Reclassifying cells, with fixed cell boundaries, or
  - Moving the cell boundaries, changing cell shape.



Original Shape

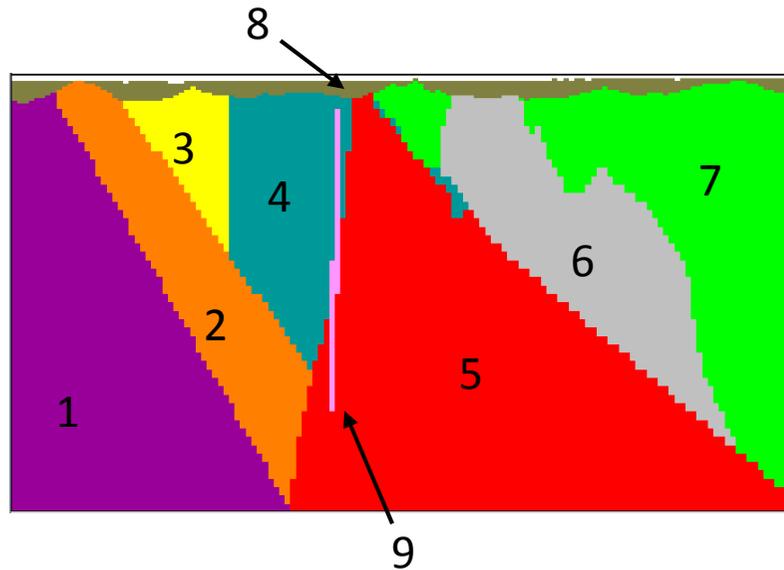


Cell defection  
(Bosch et al, 3DWEG)



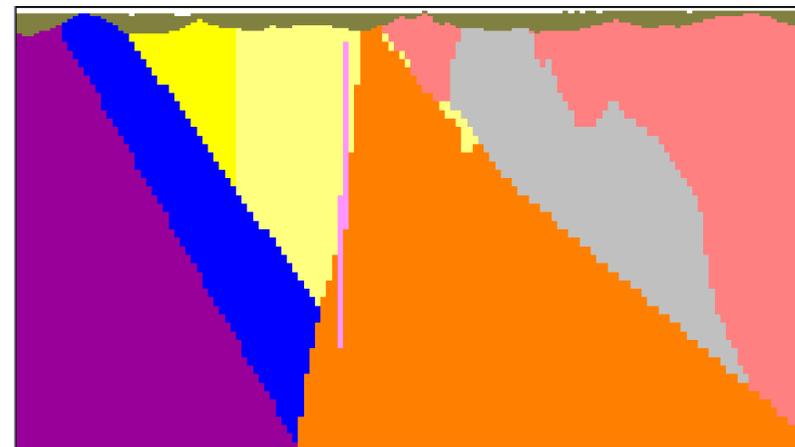
Cell deformation  
(VPmg)

# Homogeneous Property Inversion



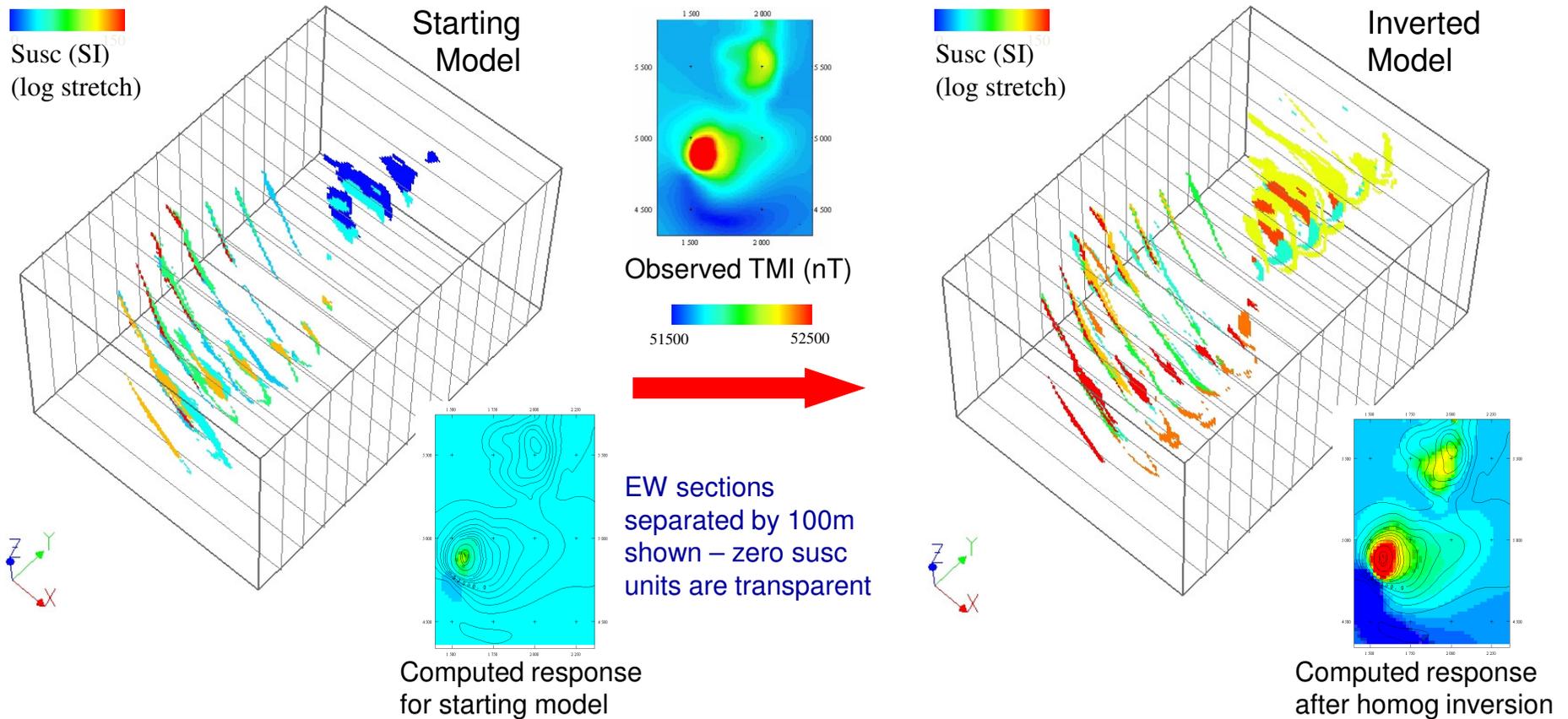
- Geometry fixed, only the properties of selected “active” units change

- Fast inversion, only a few active parameters
- Model rarely fits data after hmg property inversion

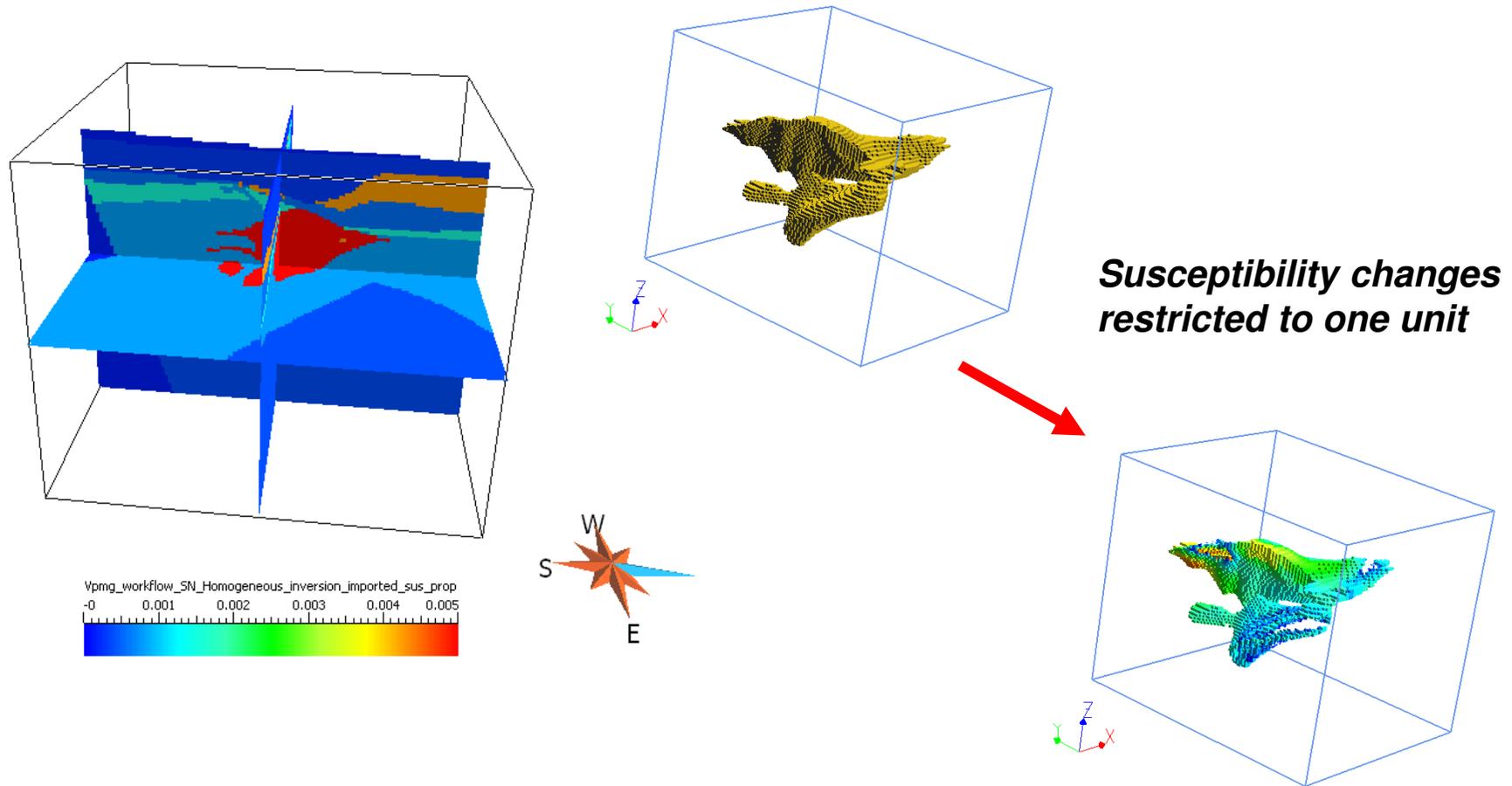


# Homogeneous property inversion

- Adjust unit properties in order to minimise misfit, with model geometry fixed

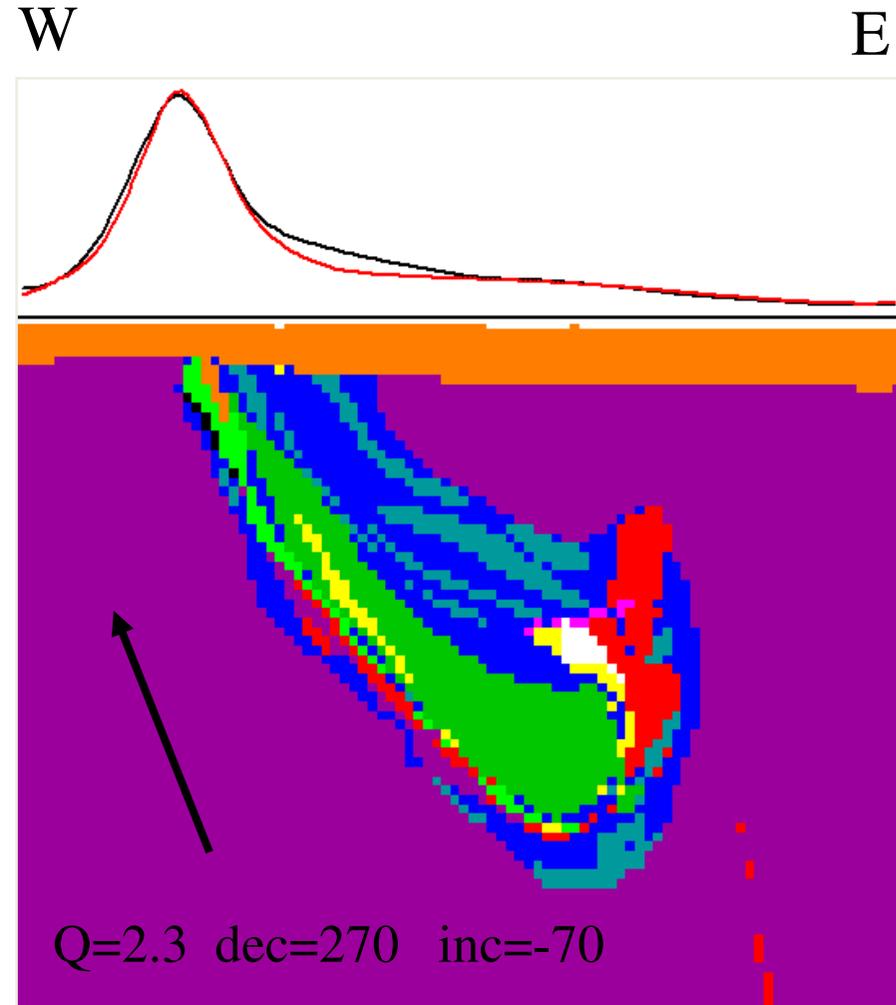
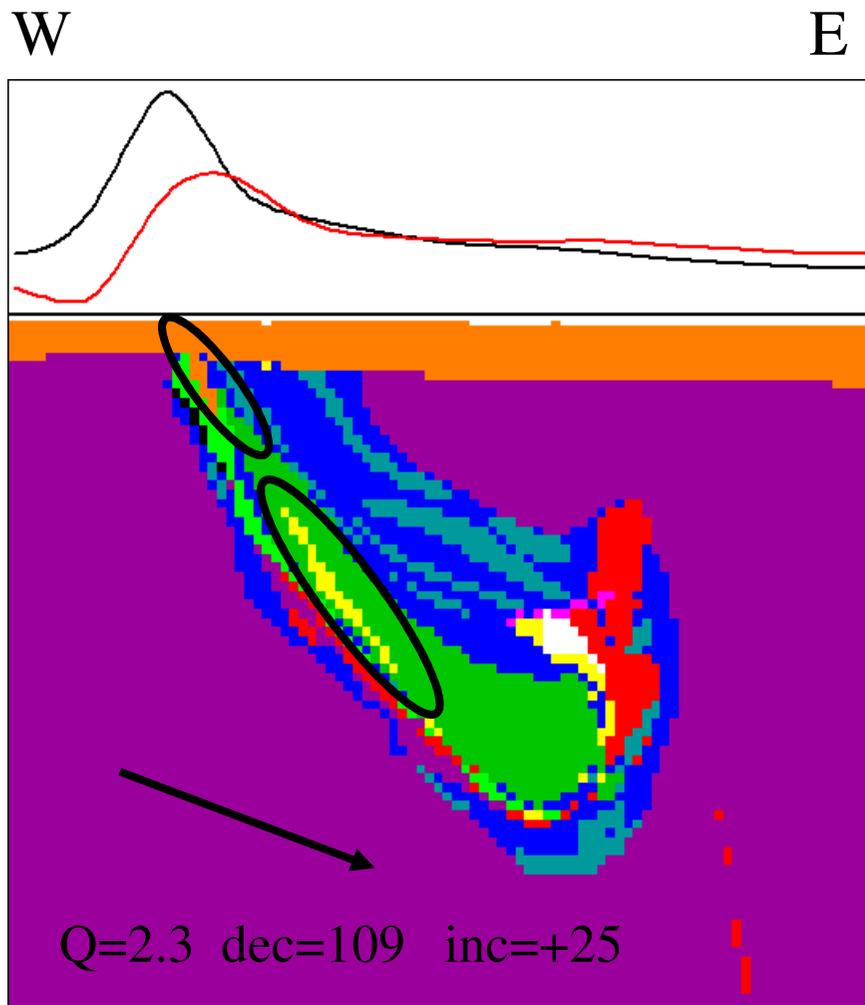


# Single heterogeneous unit inversion



**Additional control offered by geological models**

# Magnetic remanence



# Perceived disadvantages of inverting a geological model

- time required to construct model at the outset\* (though normally it either exists or not)
- time required to simplify an existing geological model
- time required to compile and analyse physical property data.

\* can be reduced by implicit geological modelling software

## Part 2

# Geological and petrophysical constraints

- Types of geological constraints
- Types of petrophysical constraints
- How many constraints make a difference?

# Geologically- and petrophysically-constrained inversion

## Purpose of geological & petrophysical constraints:

1. To restrain geologists
2. To impose ground truth (“hard” constraints)
3. To condition the inversion to emulate ground truth (“soft” constraints)



# Geologically-constrained and petrophysically-constrained inversion

- **Geologically-constrained** inversion is explicitly or implicitly constrained by geological observations.
- The intent is to preserve or favour certain geological characteristics.
- **Petrophysically-constrained** inversion honours petrophysical measurements, either explicitly or (geo)statistically or both.
- The intent is to preserve or favour certain petrophysical characteristics

# Types of geological constraints on contacts, rock types, and structures

“**Hard**” constraints fix model parameter values

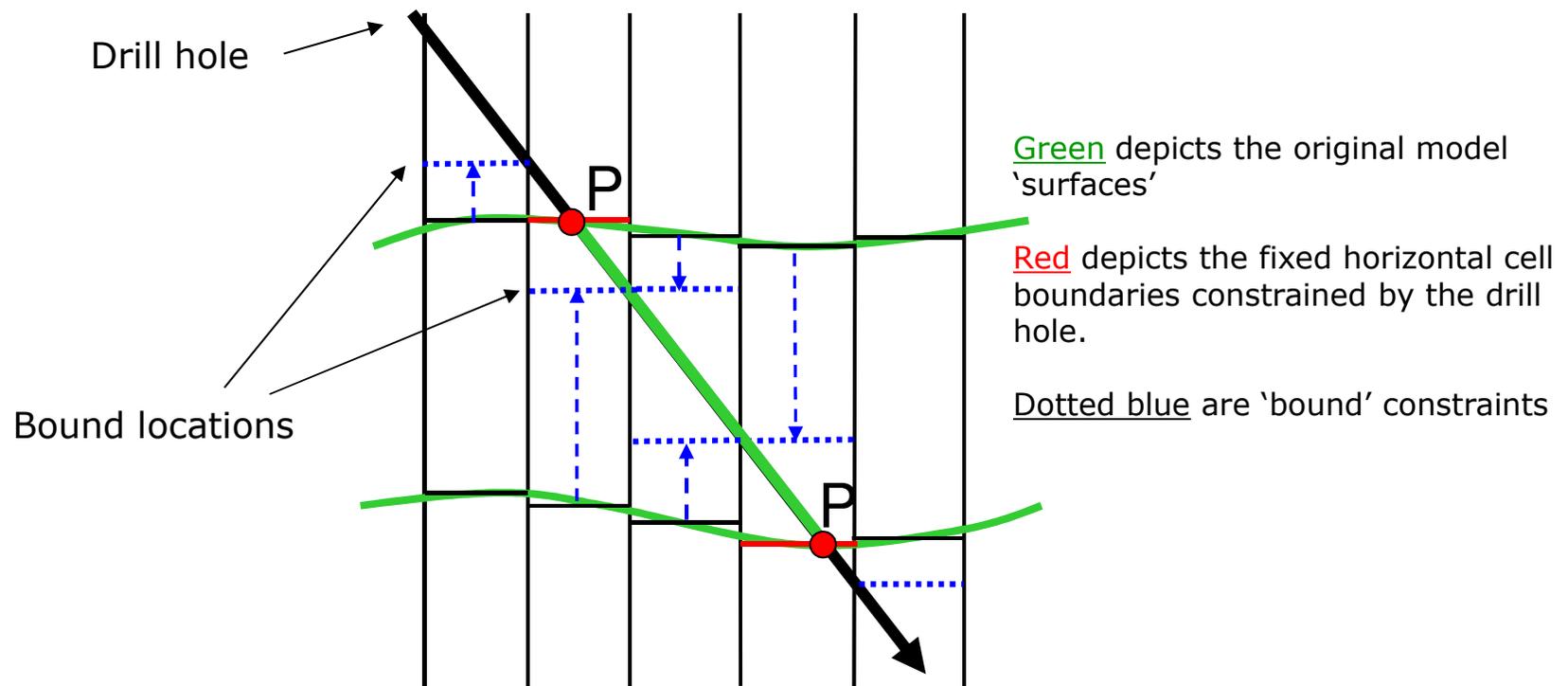
“**Soft**” constraints favour characteristics in a statistical sense

- Constraints imposing/favouring **observed** attributes are “**real**”
- Constraints imposed by user or by default are “**artificial**”

## Geometry constraints

- Fix drilled or mapped contacts during geometry inversion
- Geological logs limit changes to contact geometry
- Boundaries prevented from crossing one another
- Changes damped in neighbourhood of pierce point
- Control volume and shape of lithological domains.

# Pierce point and bound constraints in VPmg geometry inversion



# Drill hole pierce point neighbourhood of influence

- Radius of influence controls ‘derivative weighting’

Weights from nearby pierce points combined multiplicatively:

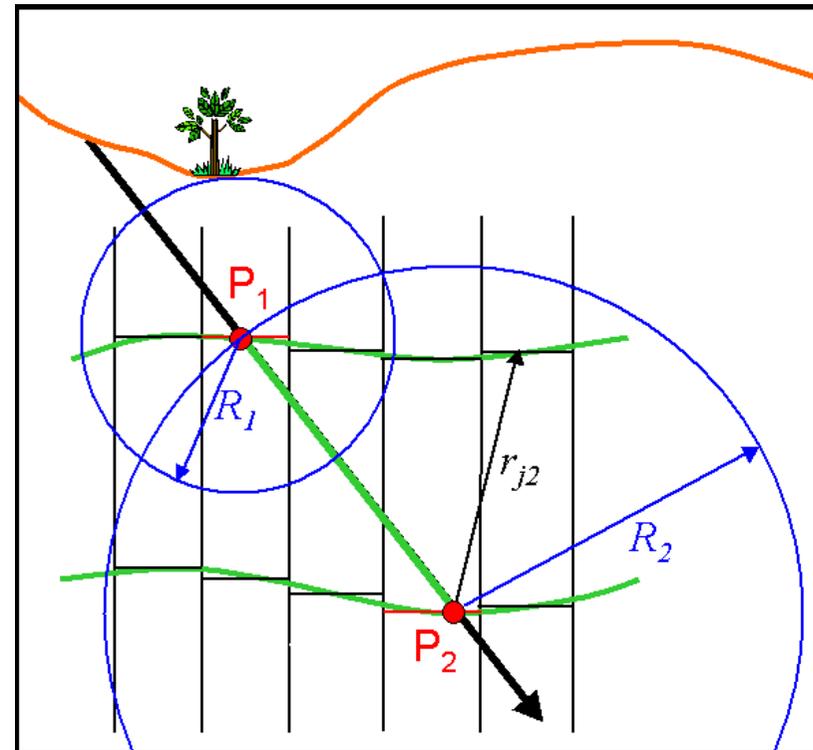
$$w_j = \prod_k \frac{r_{jk}}{R_k}$$

**Green** depicts the original model surfaces

**Black** depicts the horizontal and vertical cell boundaries of the VPmg model

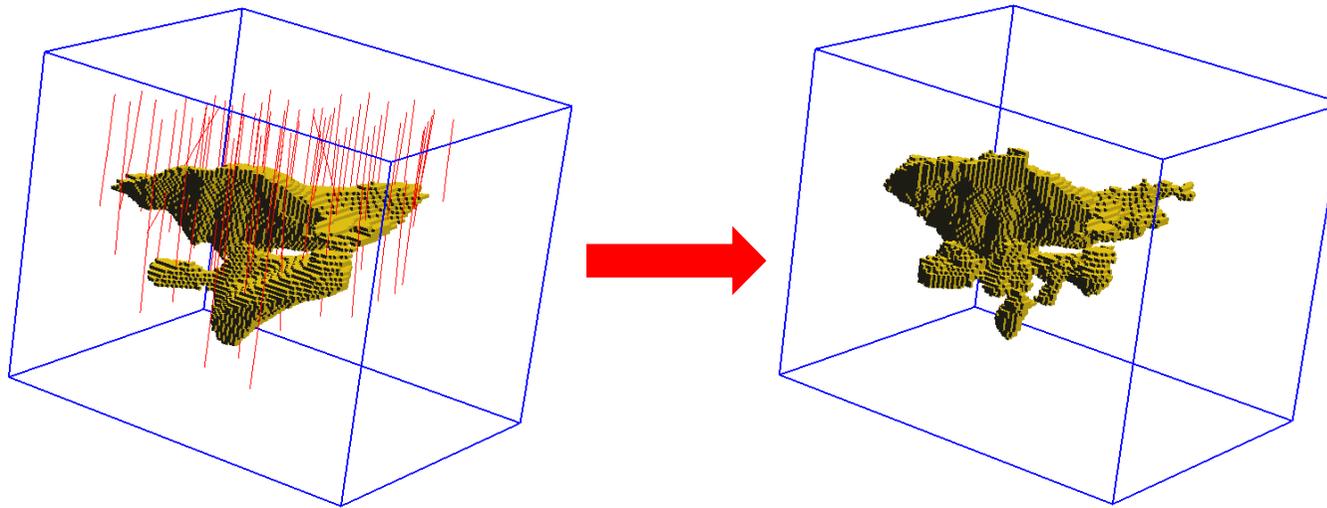
**Red** depicts the fixed horizontal cell boundaries constrained by the drill hole.

**Blue** conceptually illustrates the radius of influence about pierce points.



# Constrained geometry inversion of a sulphide body

Example of geological non-uniqueness



**Original** interpretation

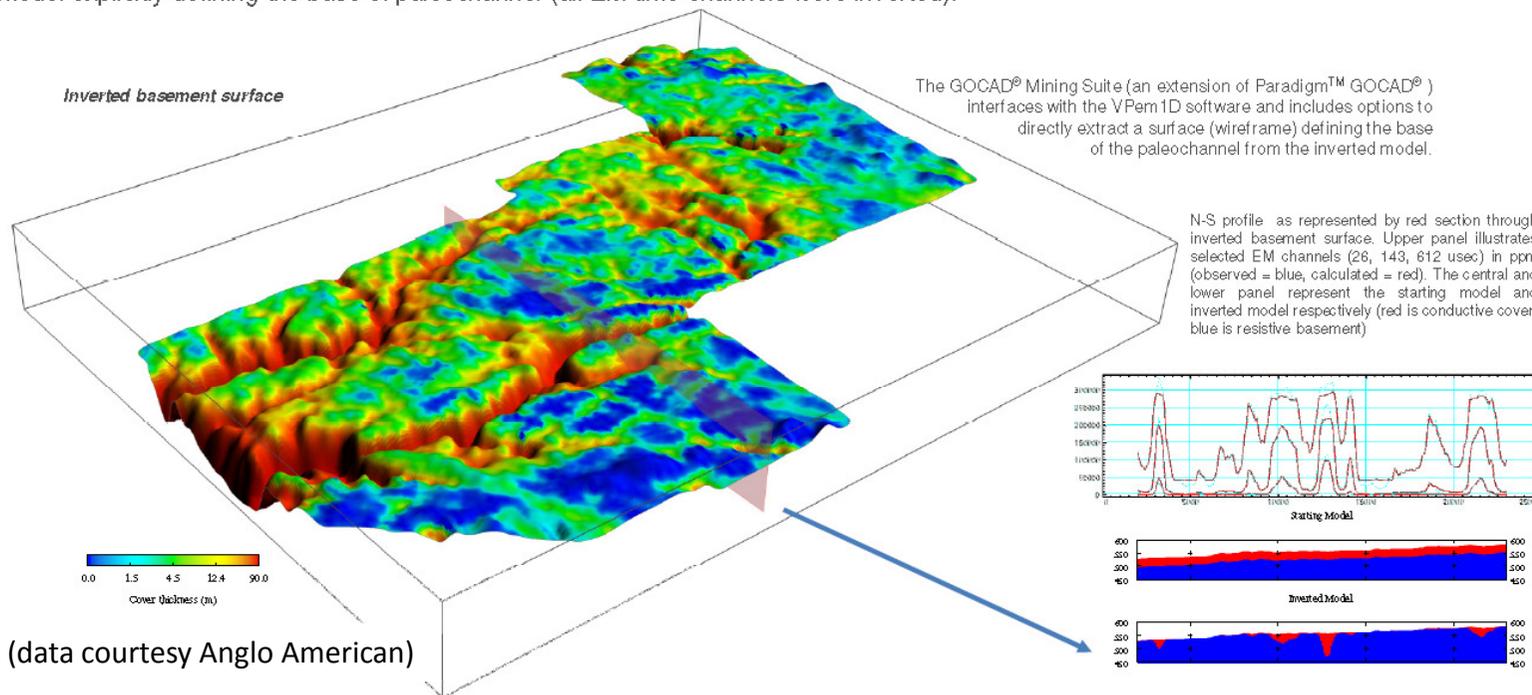
**Revised** interpretation,  
after magnetic (geometry) inversion,  
honours the same drill hole pierce points

# Inversion of a geological model

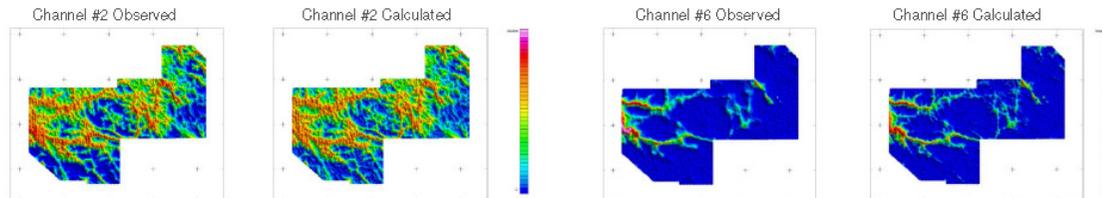
## Geometry inversion of Spectrem over a paleo-channel: underlying model is geological

### Spectrem Example:

A simple two layer starting model was assumed comprising 30m of conductive (100mS/m) cover/paleochannel overlying resistive (1mS/m) basement. VPem1D geometry inversion adjusted the base of the upper conductive layer to produce a simple geological model explicitly defining the base of paleochannel (all EM time channels were inverted).



Geometry inversion adjusted the base of the paleochannel reducing the chi-square data misfit from 8436 to 55 (assuming a 1% data uncertainty). Data misfit could be further reduced by then inverting for conductivity variations in the basement (or cover). Observed and calculated responses of selected EM time channels (#2 @65 usec and #6 @1237 usec) are shown in map view to the right.



(Mira Geoscience & Fullagar Geophysics)

# Types of geological constraints cont'd

## Stratigraphic constraint

- Enforce stratigraphic relationships

## Structural constraints

- Structural constraints on dip, strike, plunge
- Favour lateral continuity, depth range, &/or shape
- Enforce commonality of gradients in joint inversion

## Reference model constraint

- Restrict deviations from starting (or “reference”) model

# Bosch inversion 2D example (c.f. 3DWEG)

Permit changes to properties of cell, its lithotype, & its vertices

Set of lithotypes fixed

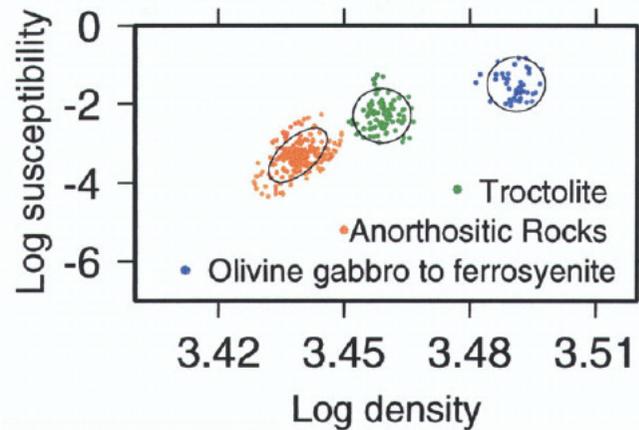
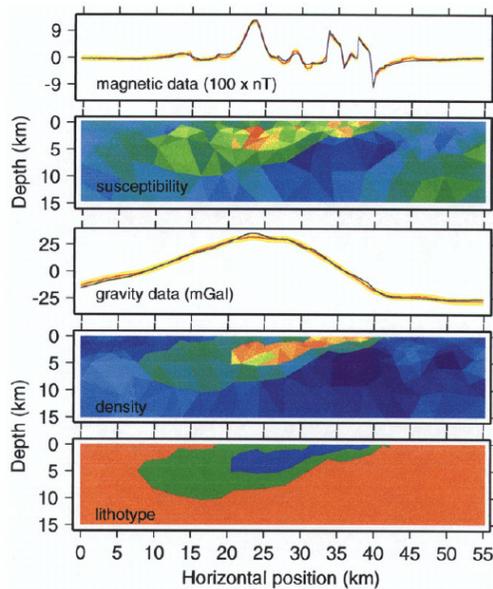
Lithotype map at Earth's surface is known/fixed

Density & susceptibility honour prior distributions

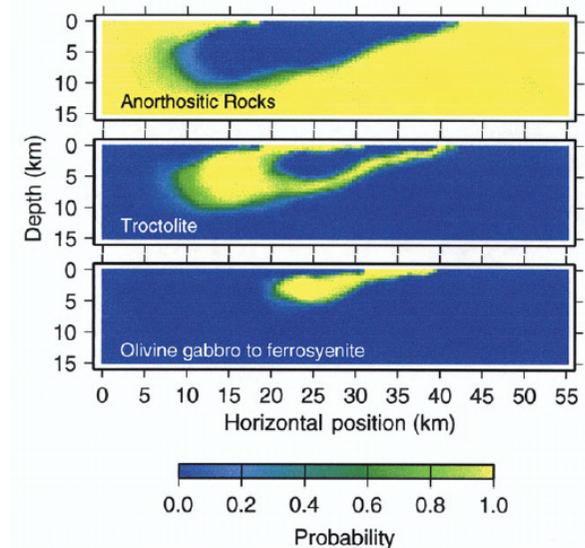
Multiple regions with same lithotype; volume fraction; area/perimeter?

**Stratigraphic constraints**, e.g. olivine gabbro not permitted to make direct contact with anorthositic host rocks

Example: Kiglapait, Labrador

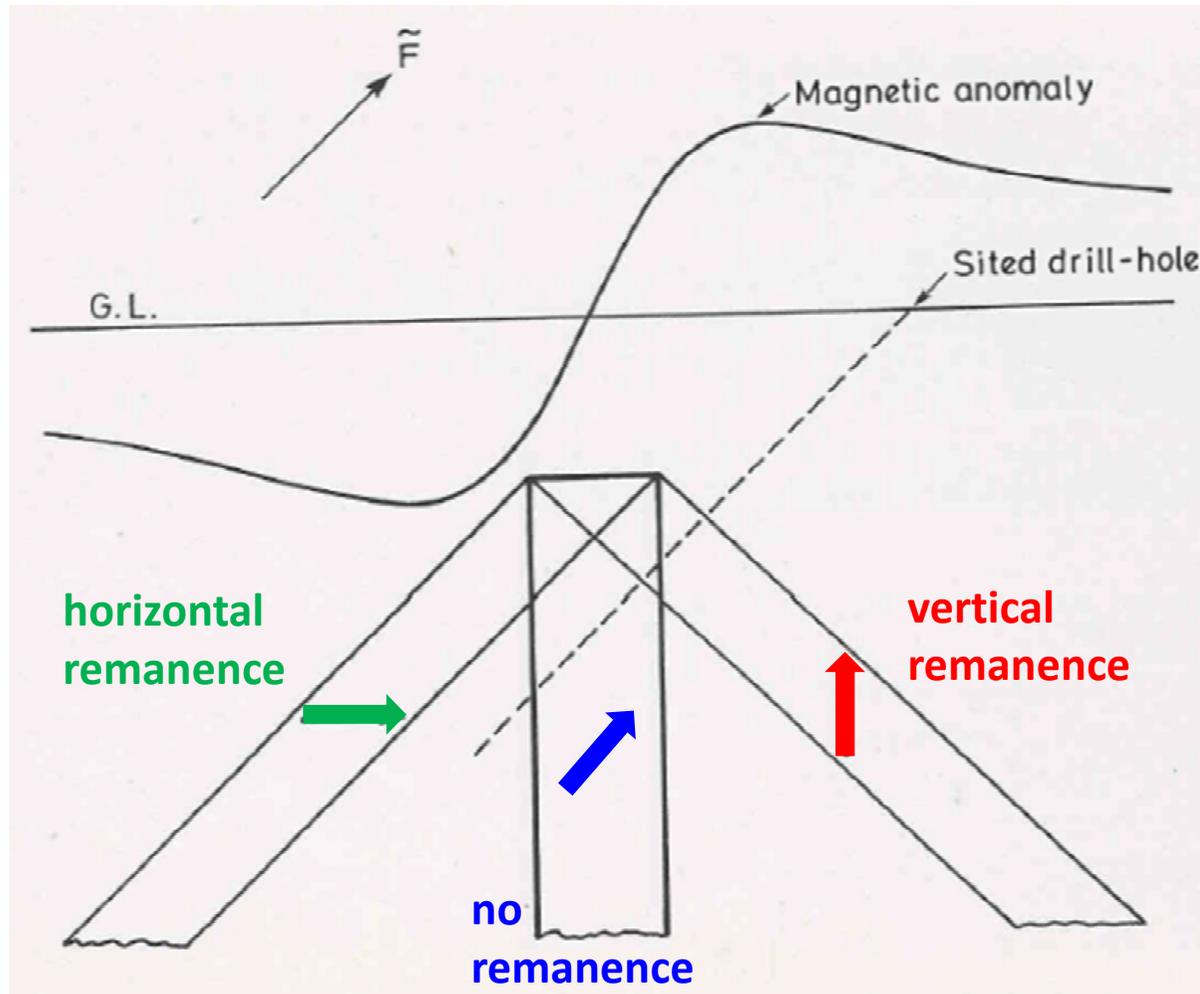


(after Bosch & McGaughey, 2001)

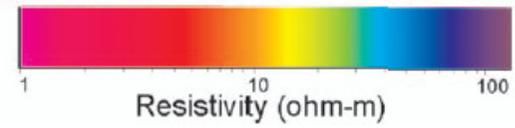
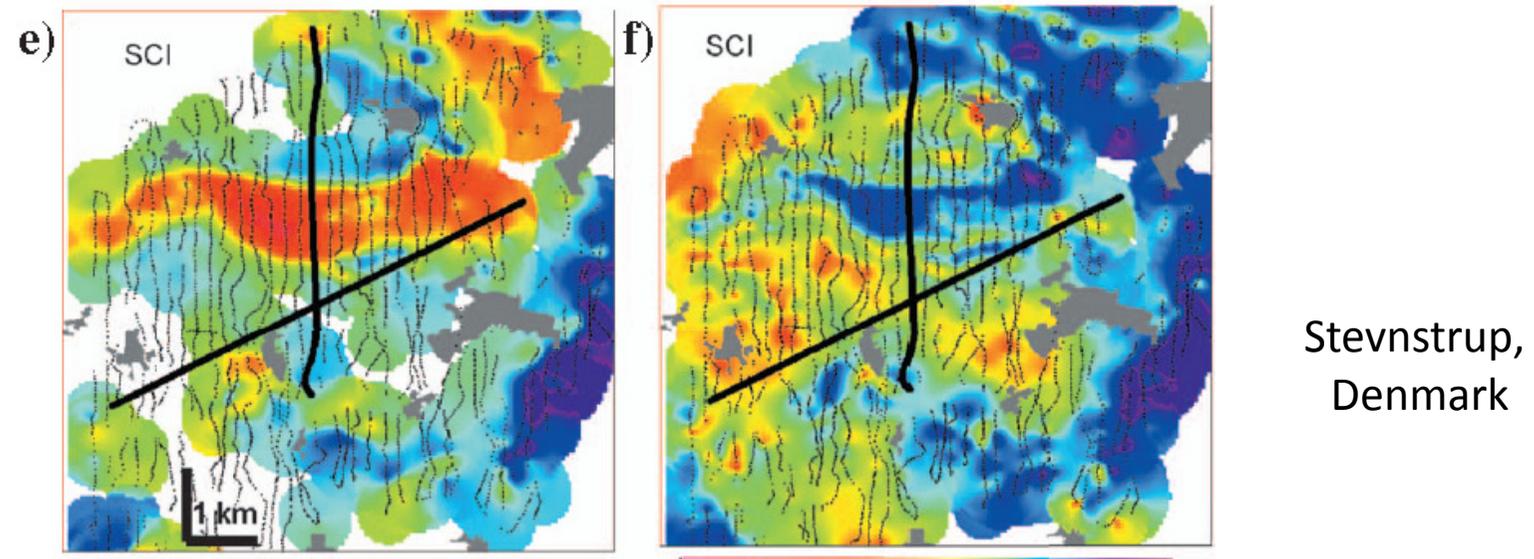
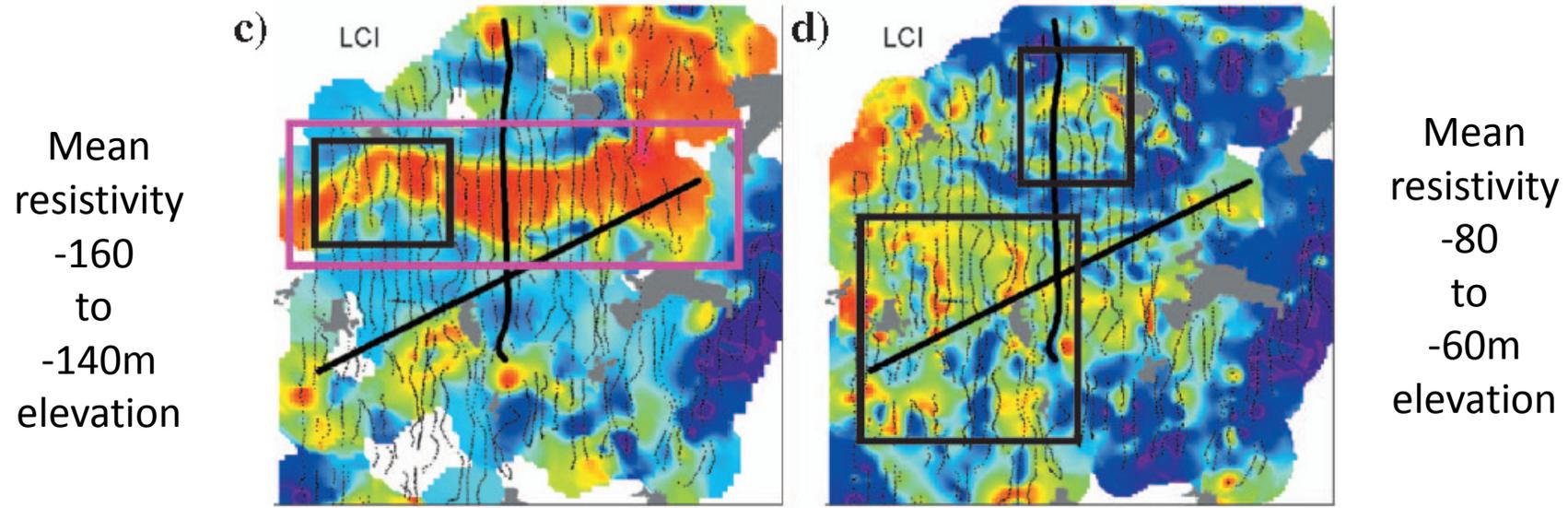


# Non-uniqueness of magnetisation inversion

Can be resolved in this case if dip is known



# Laterally and spatially constrained 1D TEM inversion



## Joint inversion with “structural resemblance” (Gallardo et al, 2012)

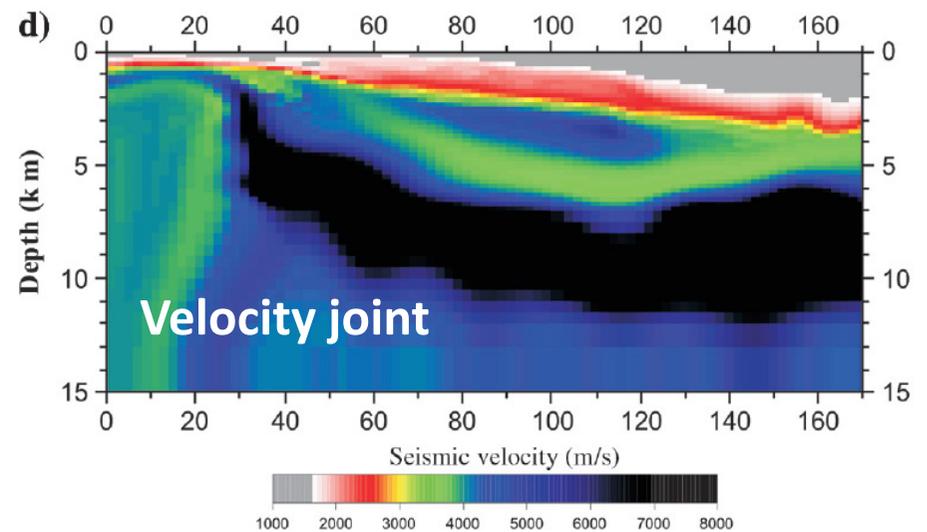
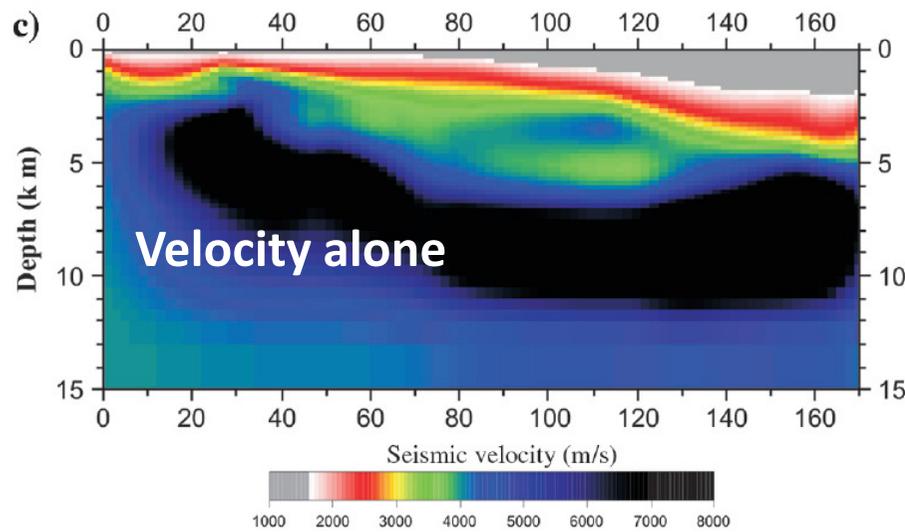
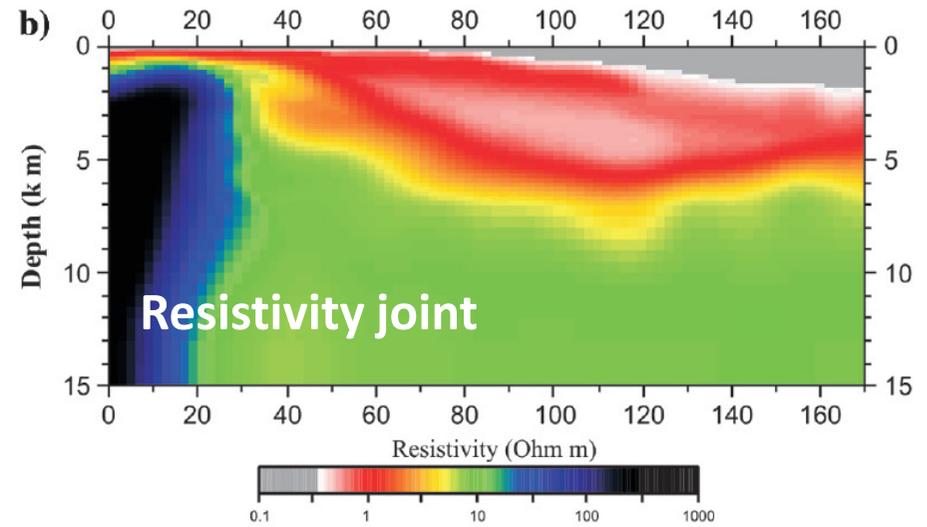
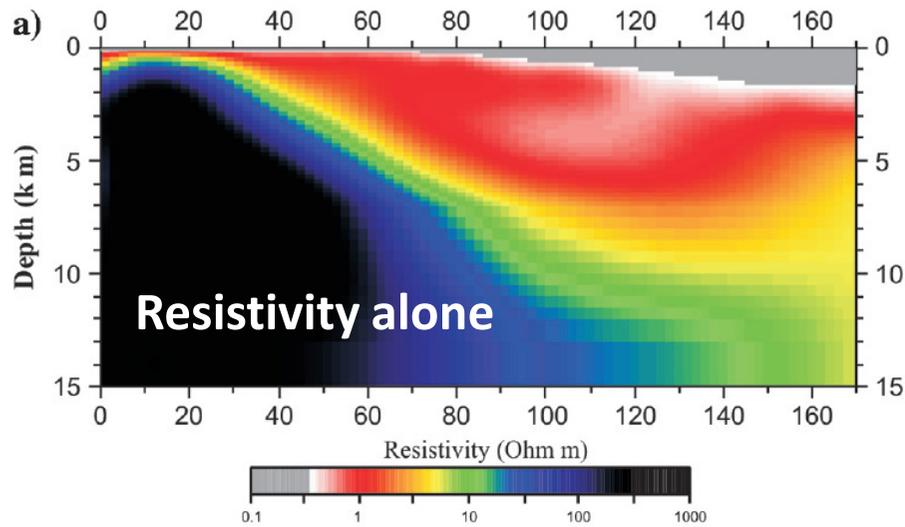
based on the principle of structural resemblance as measured by the crossgradient function  $\vec{\tau}$  (Gallardo and Meju, 2003) given by:

$$\tau(\mathbf{m}_i, \mathbf{m}_j) = \nabla m_i(x, y, z) \times \nabla m_j(x, y, z), \quad (1)$$

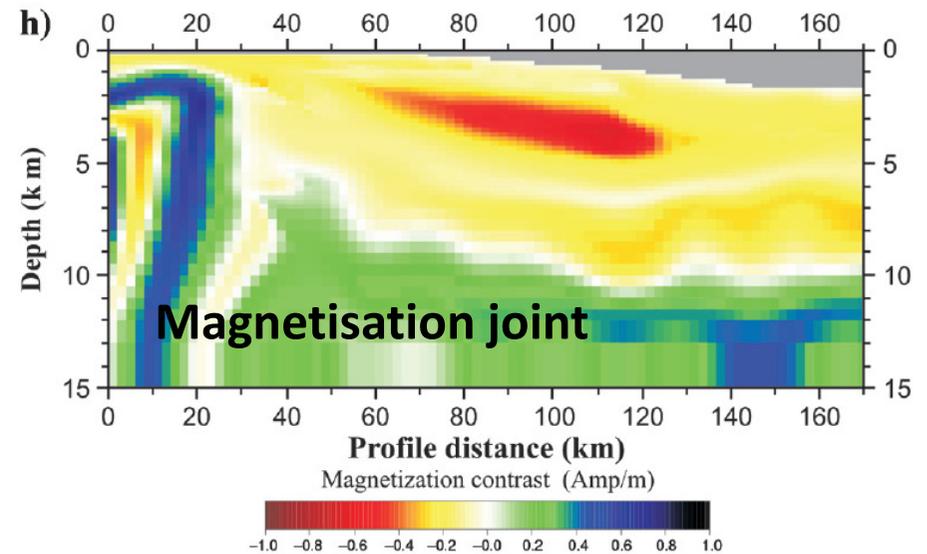
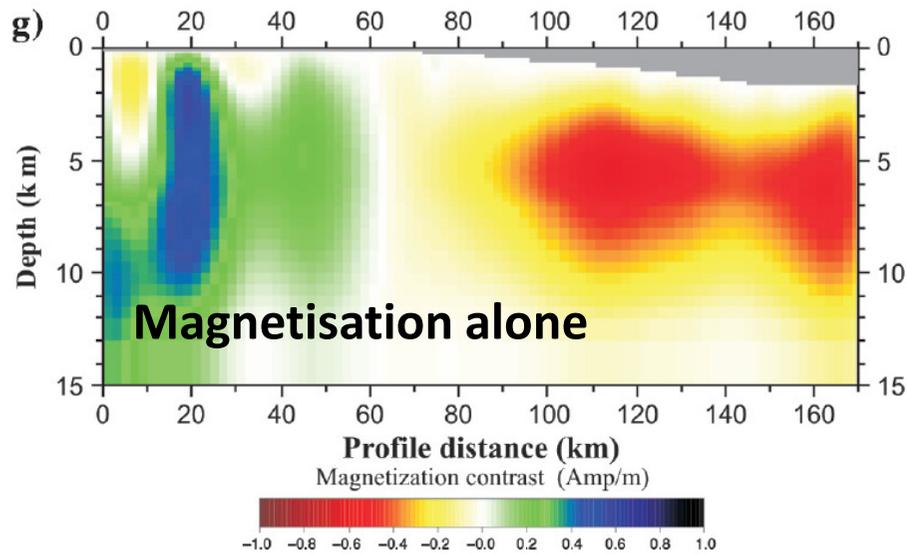
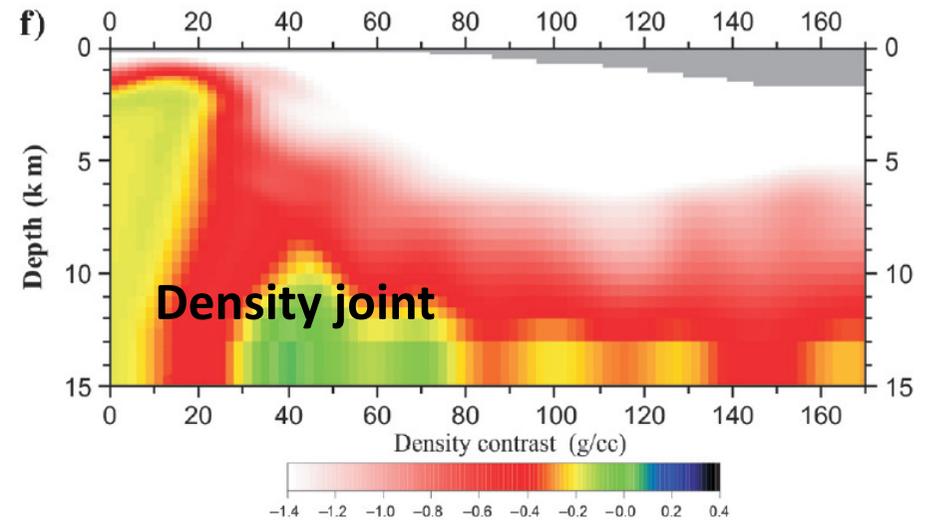
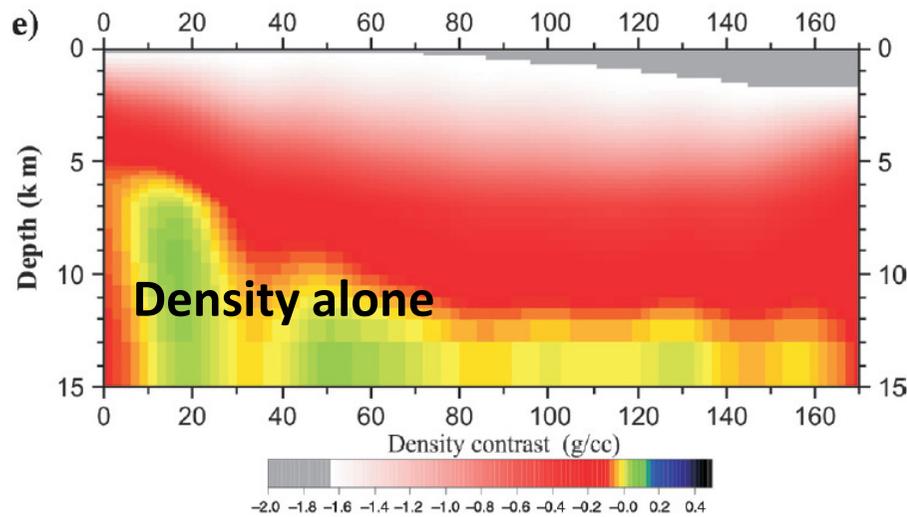
where  $\mathbf{m}_i$  represents the set of parameters for the  $i$ th physical model.

- Condition  $\tau(\mathbf{m}_i, \mathbf{m}_p) = \mathbf{0}$ , incorporated in objective function for joint inversion
- Therefore, property gradients must be parallel => “common boundaries”  
or else at least one gradient must be zero => local homogeneity
- A reasonable approach, but “commonality” not guaranteed in the real Earth.

## 2D joint inversion with “structural resemblance” (Gallardo et al, 2012)



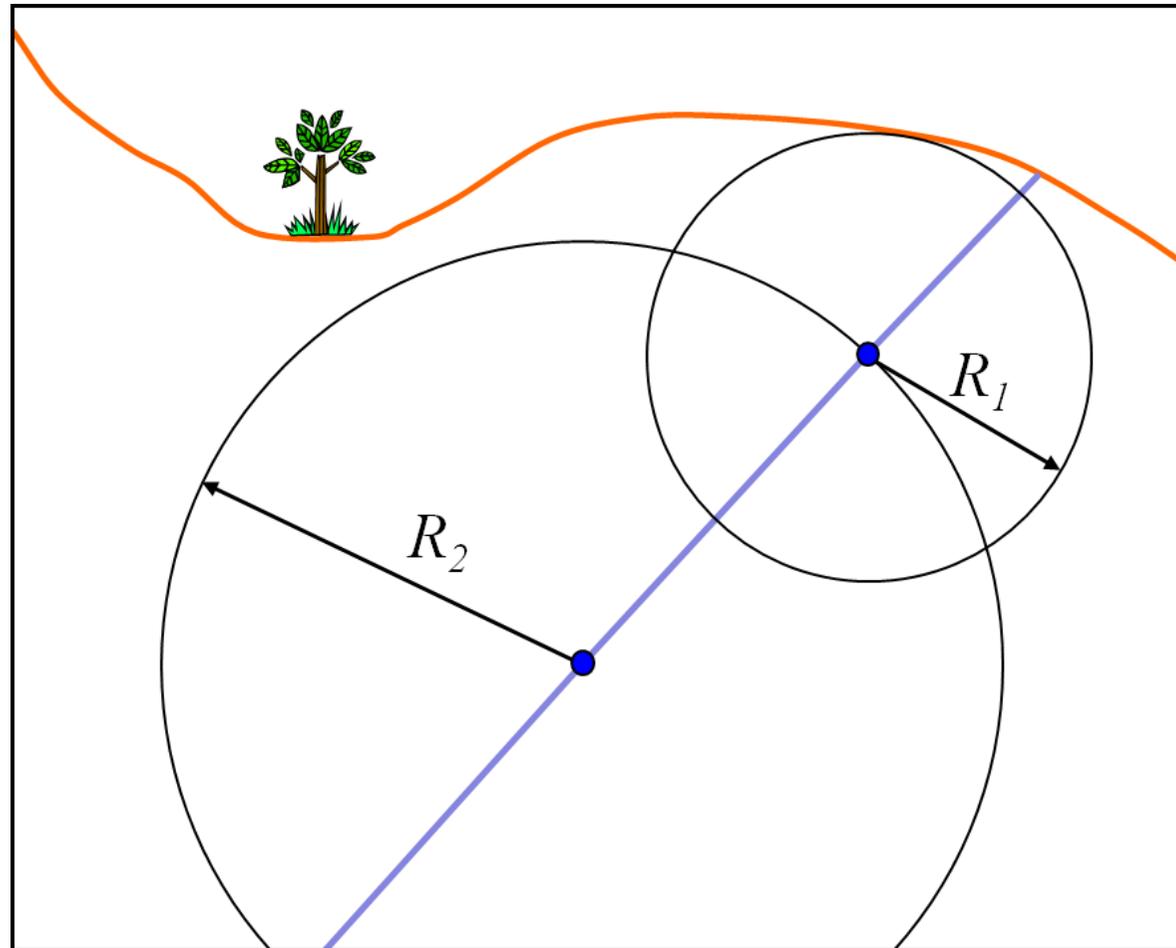
## 2D joint inversion with “structural resemblance” (Gallardo et al, 2012)



# Types of petrophysical constraints

- upper and lower property bounds
- fixed property cells, with neighbourhood of influence
- conditioning, e.g. smoothness, compactness
- statistical and geostatistical constraints
- Lithology inferred from inverted properties, using either supervised or unsupervised methods.

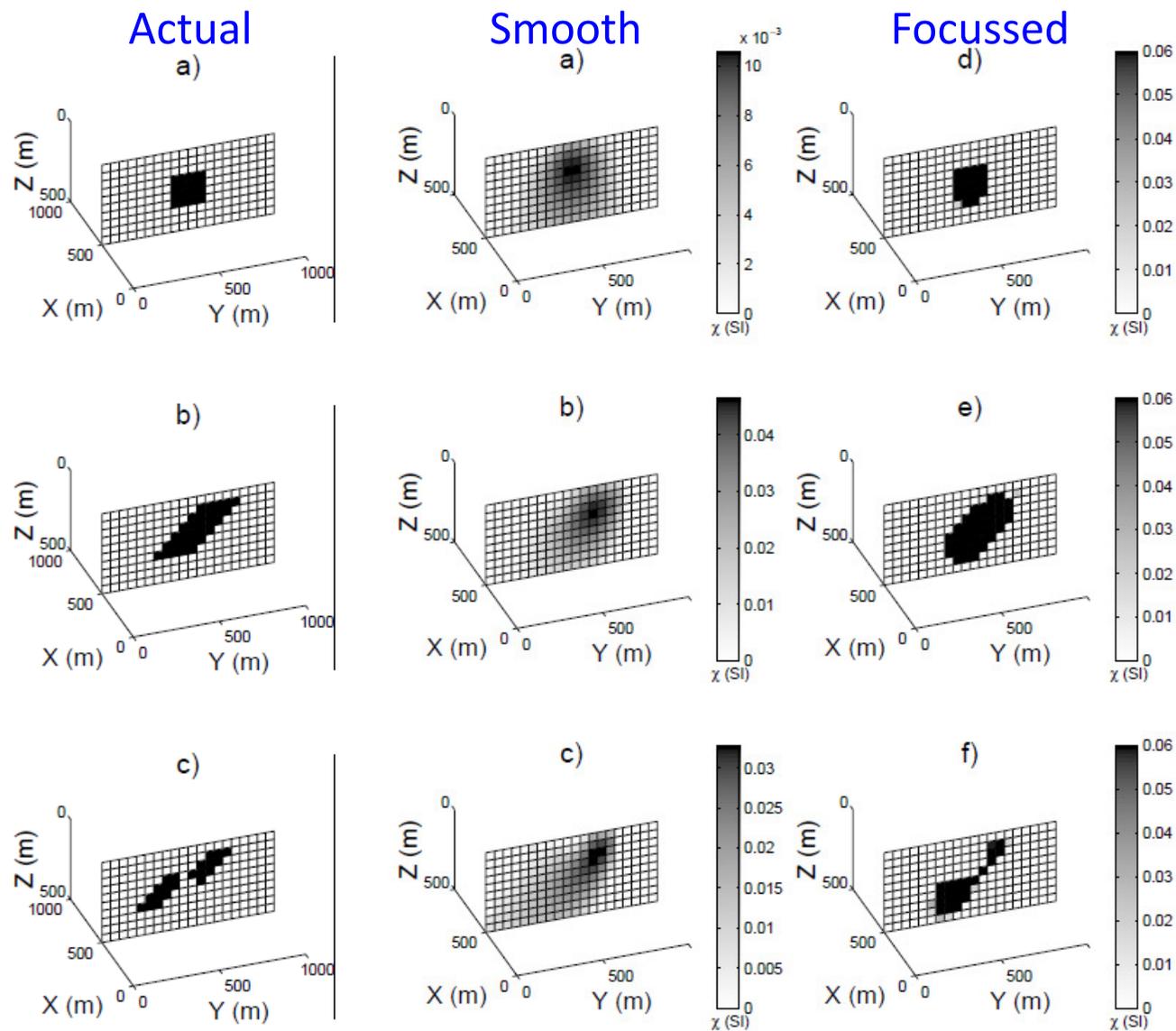
## Fixed property cells, with neighbourhood of influence



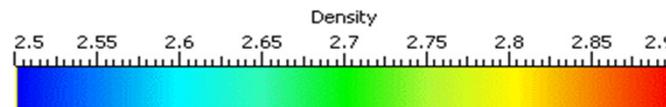
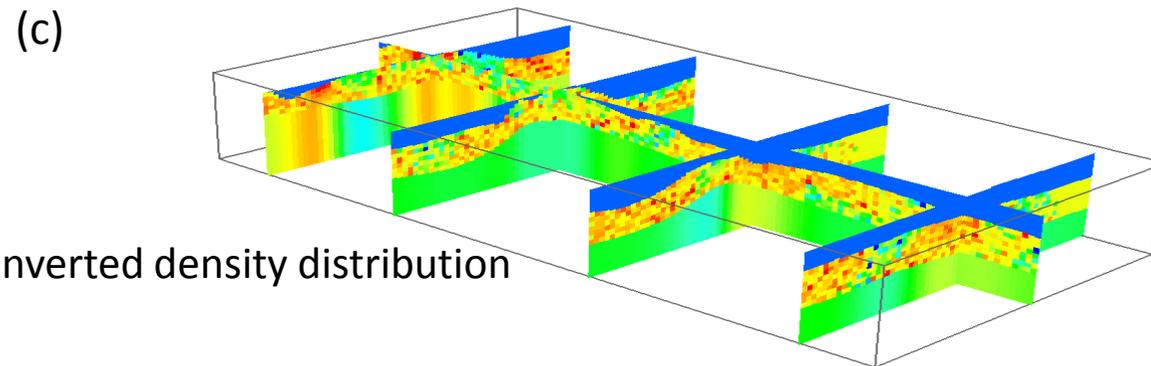
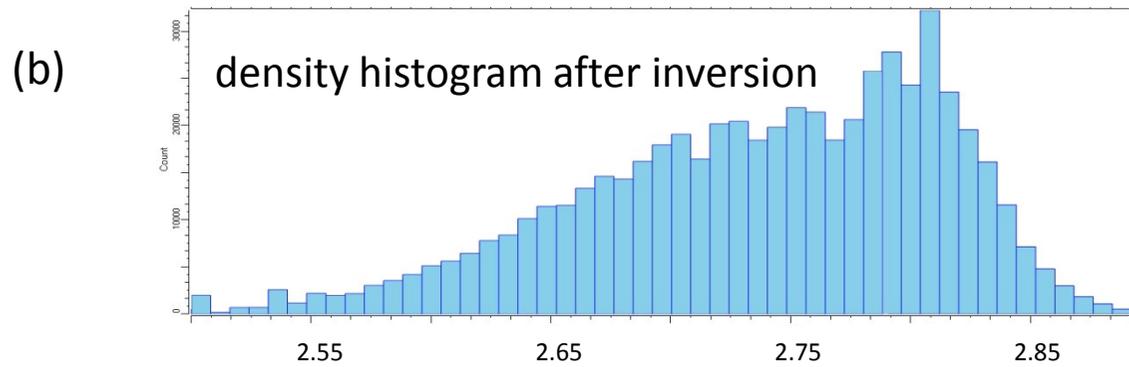
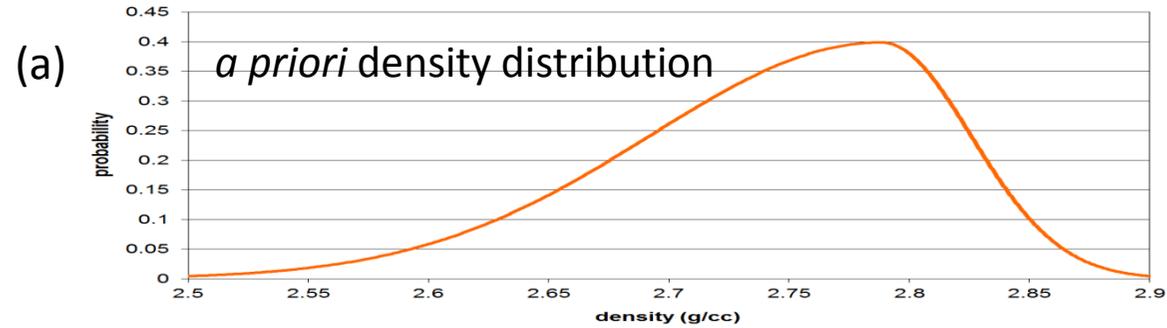
**Avoid “string of beads”**

# Focusing inversion of magnetic data

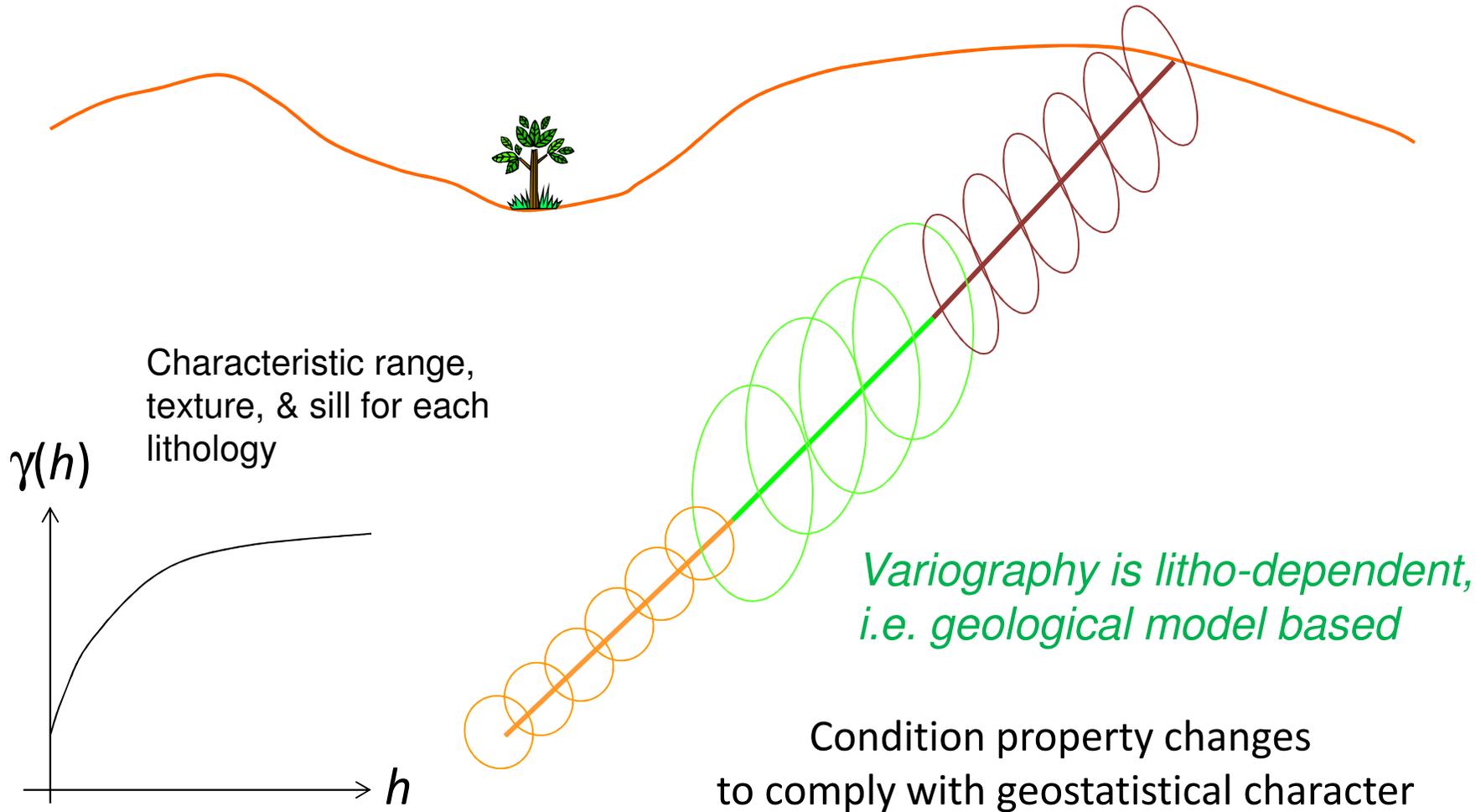
(Portniaguine & Zhdanov, 2002)



# Stochastic inversion of gravity over a limestone

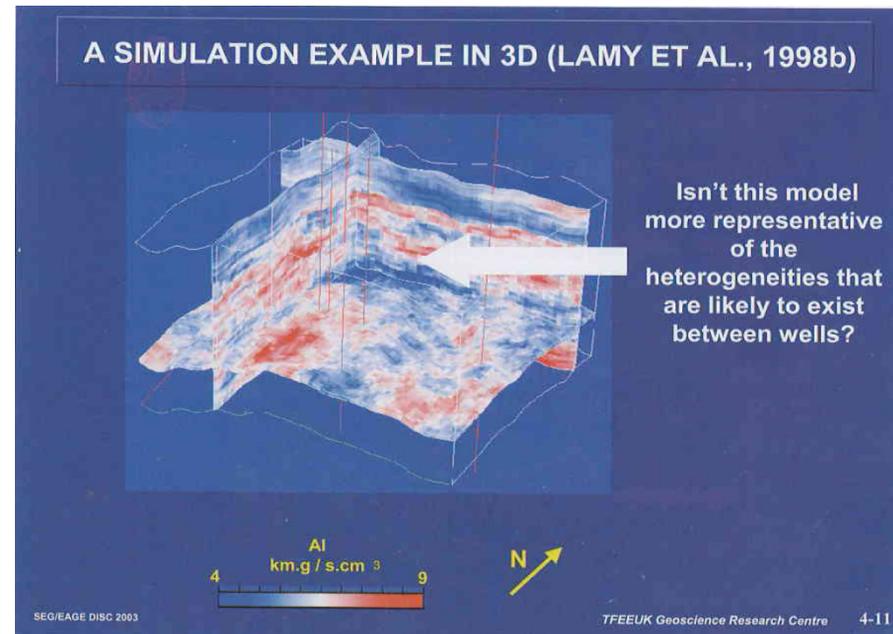
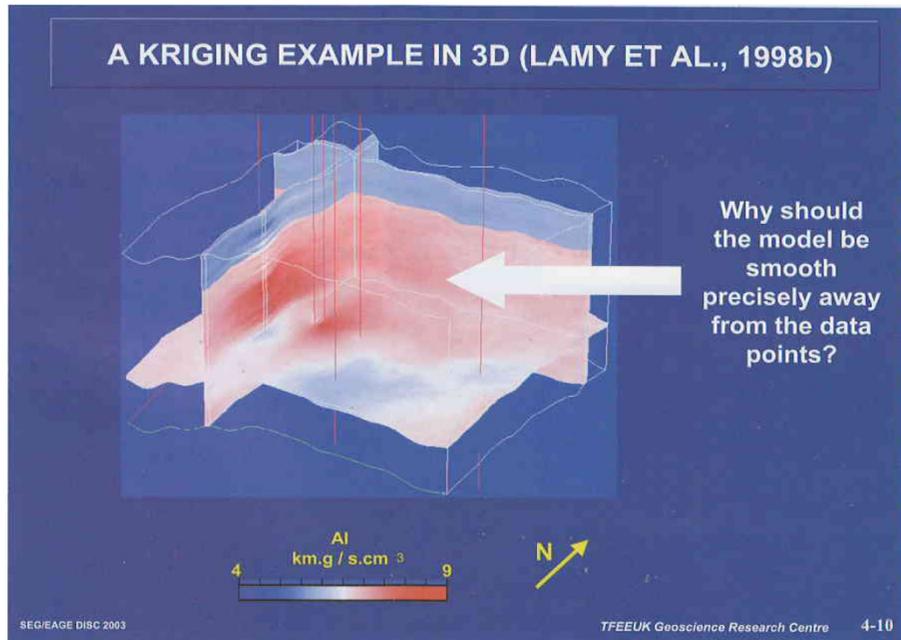


# Geostatistical constraints



# Stochastic simulation versus kriging

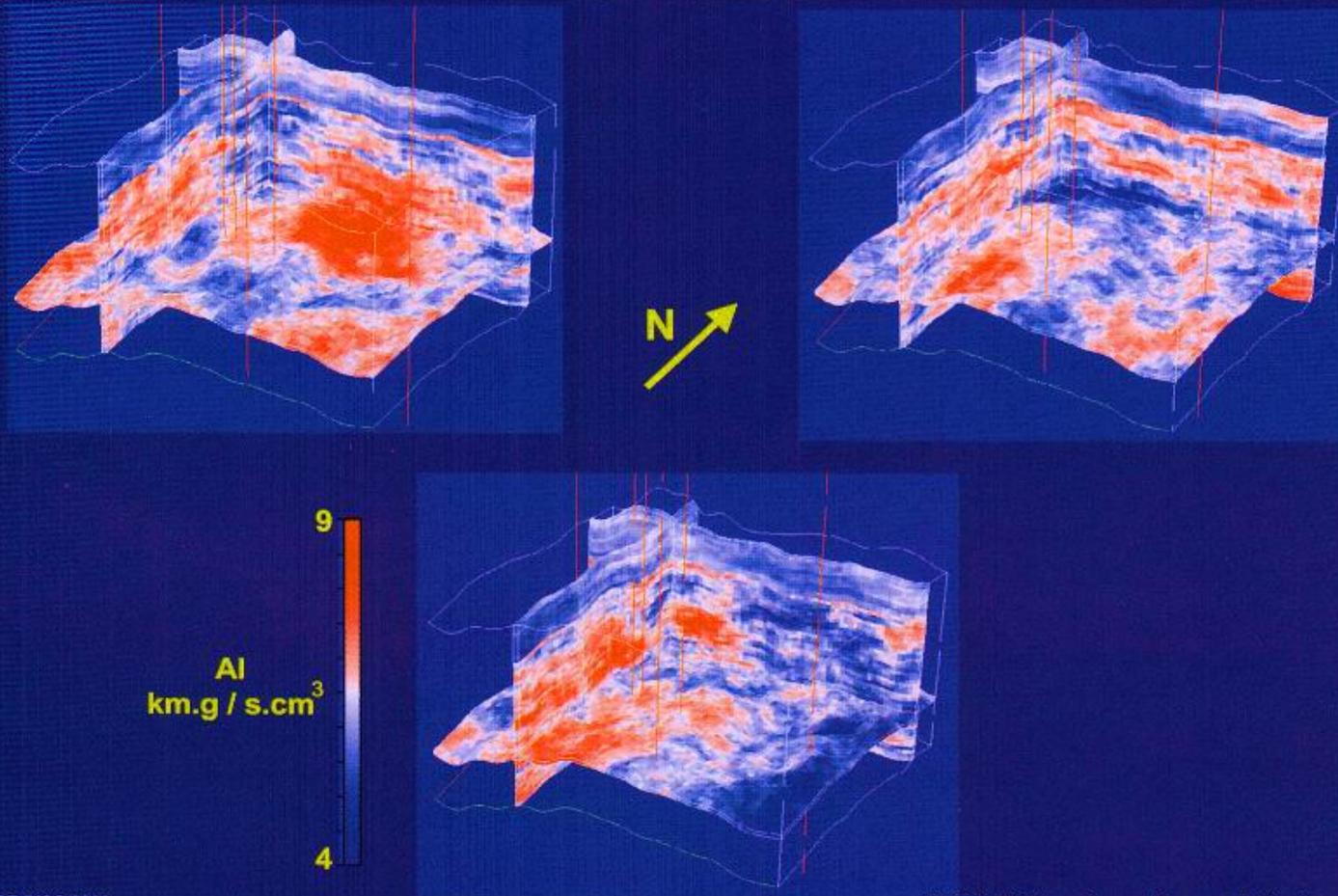
Acoustic impedance models, controlled by seismic and wireline logs



(Dubrule, 2003)

# GENERATION OF MULTIPLE REALIZATIONS

## Acoustic impedance distributions

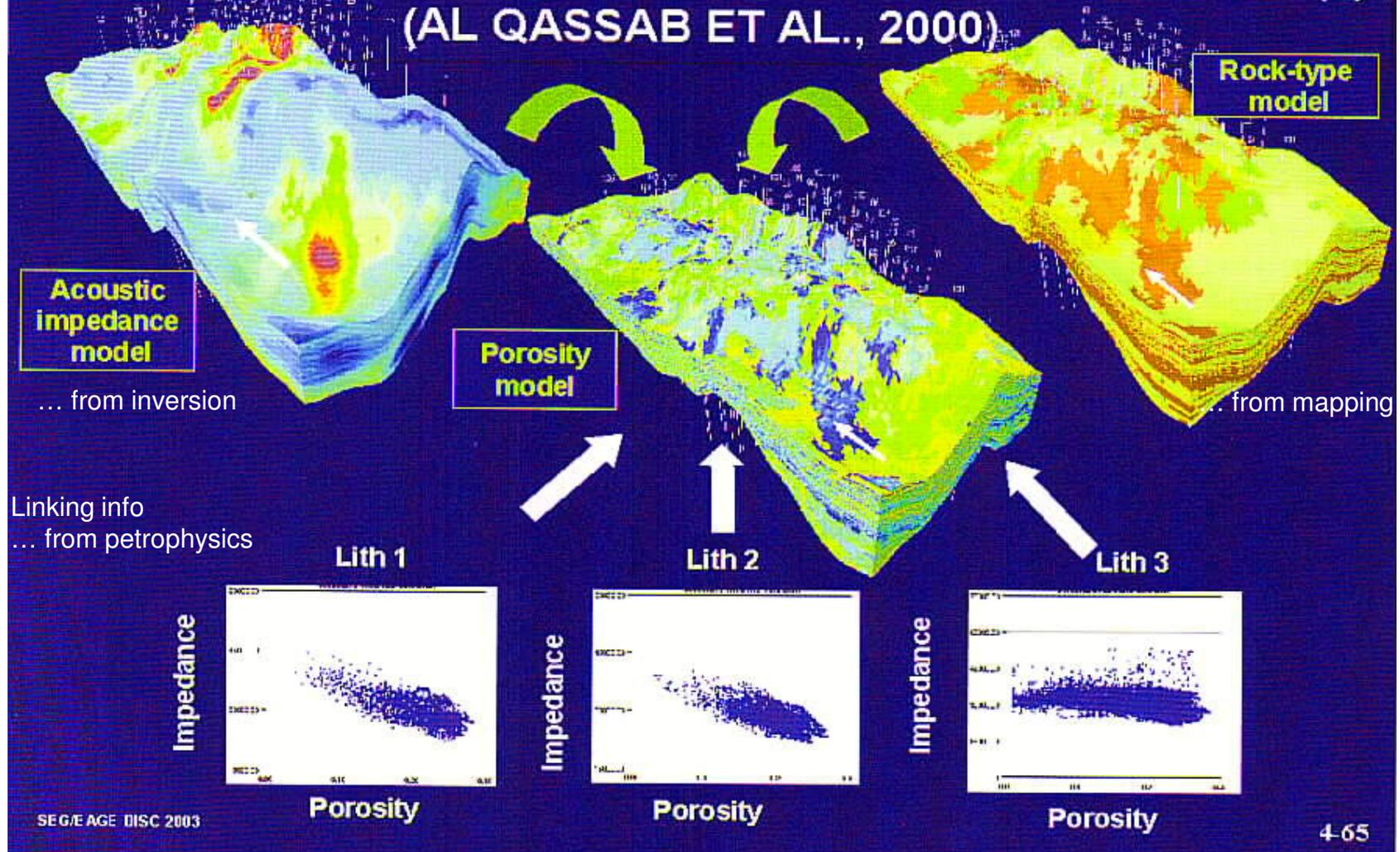


SEG/EAGE DISC 2003

TFEEUK Geoscience Research Centre

4-12

# SEISMIC AND FACIES-BASED POROSITY MODEL, UNAYZAH RESERVOIR, HAWTAH FIELD, SAUDI ARABIA (3) (AL QASSAB ET AL., 2000)



(after Dubrule, 2003)

# Bosch inversion 2D example (c.f. 3DWEG)

Permit changes to properties of cell, its lithotype, & its vertices

Set of lithotypes fixed

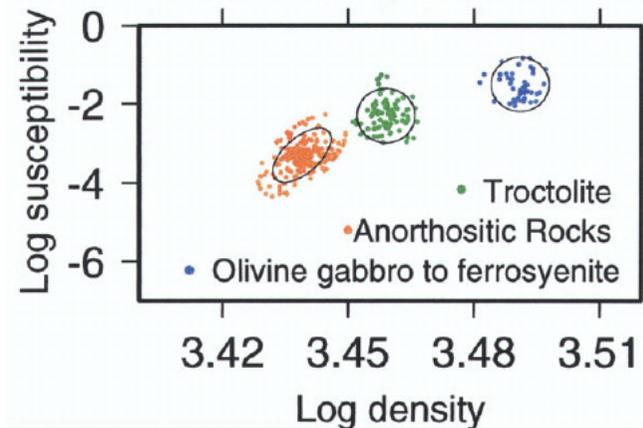
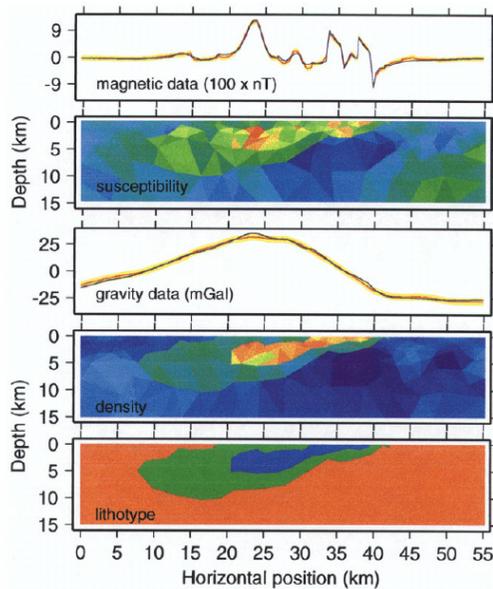
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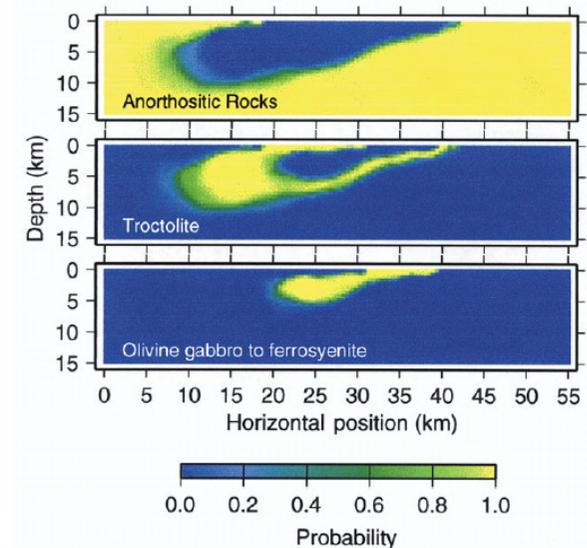
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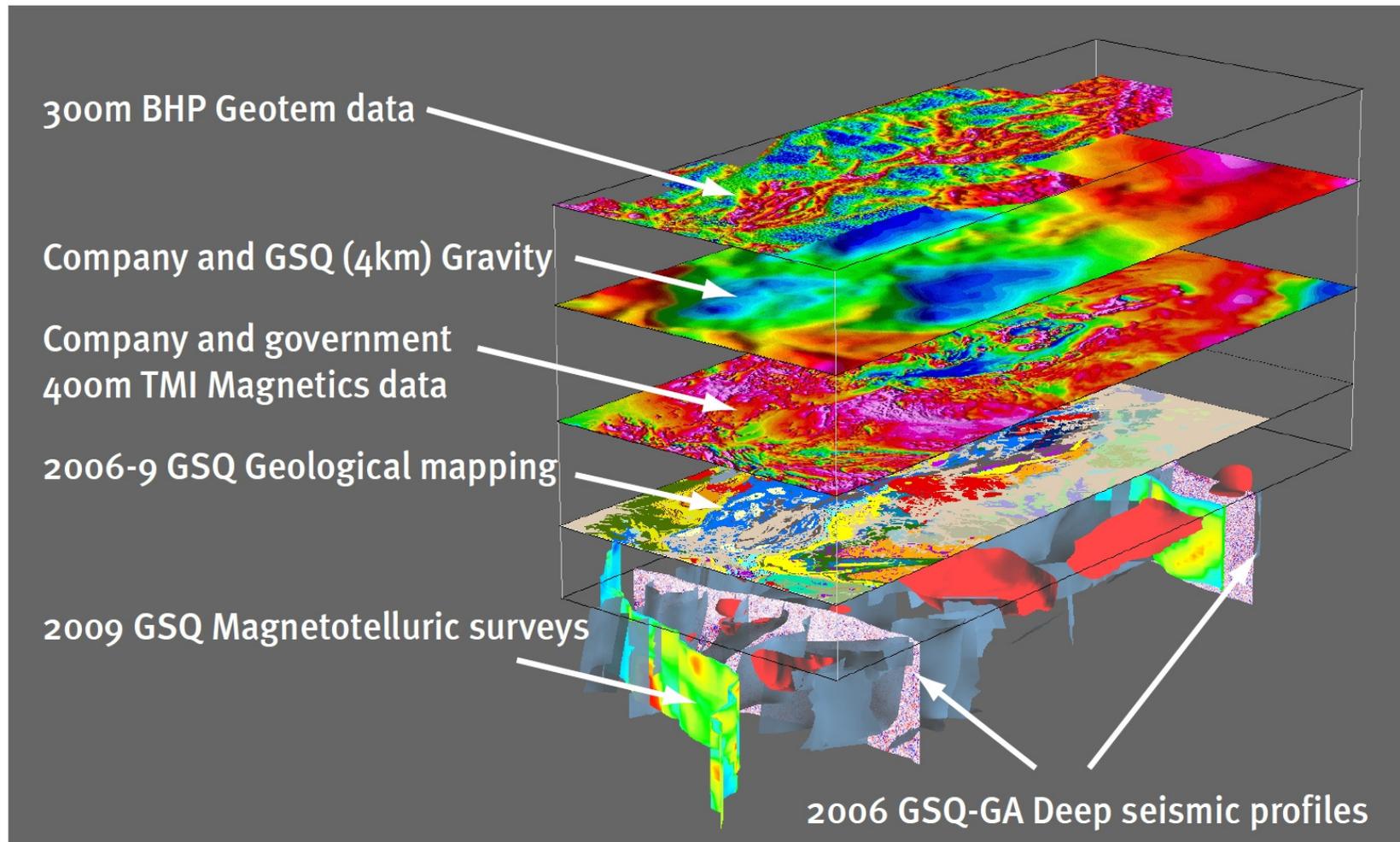
(after Bosch & McGaughey, 2001)



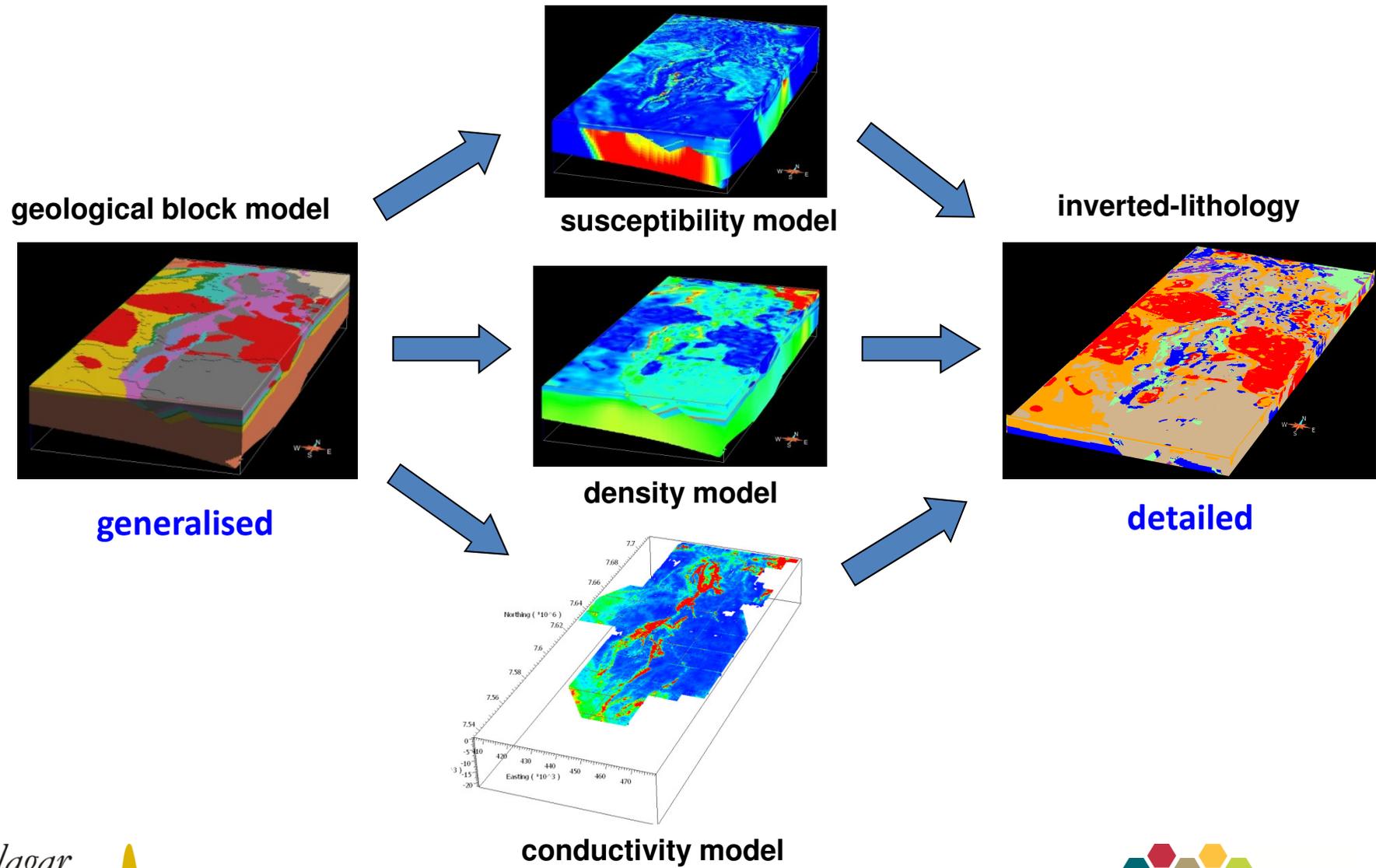
# Interpret lithology from multiple data sets

Mount Dore integrated interpretation, for Geological Survey of QLD

Main components: AEM, gravity, magnetics, and geological mapping



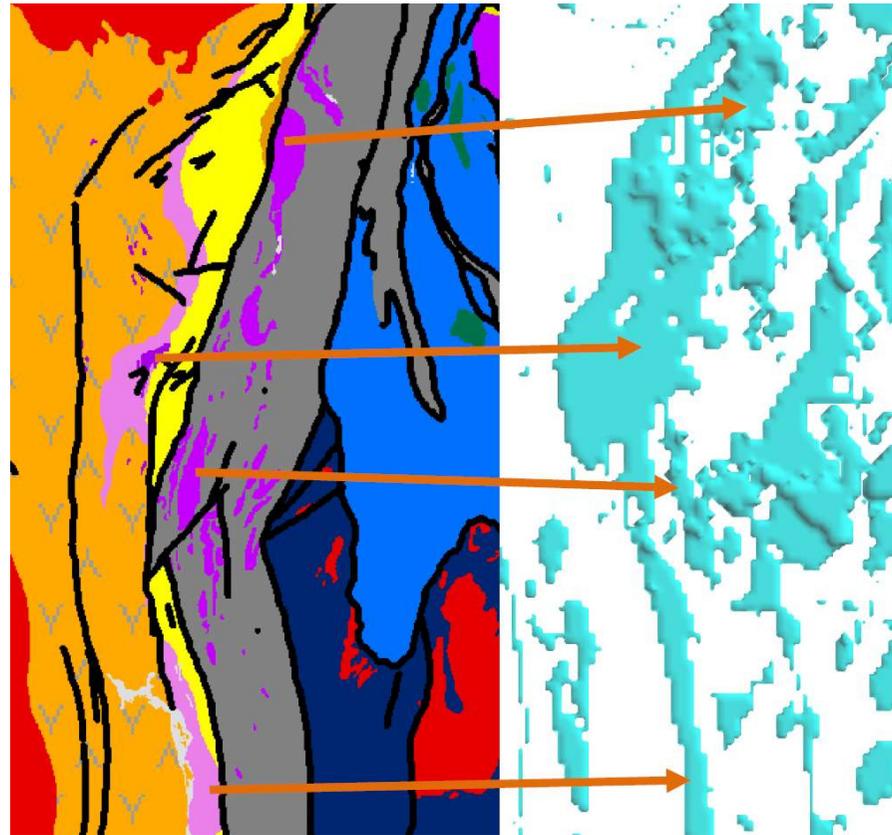
# Mount Dore inversion & inverted-lithology workflow



# Mount Dore inverted-lithology

Inverted facies and initial facies agreed in 72% of cells.

Regions reclassified by *LogTrans* as Double Crossing Metamorphics (high density and high susceptibility) are coincident with mapped intrusives which were not included in the original geological model.



# How many constraints make a difference?

**It depends!**

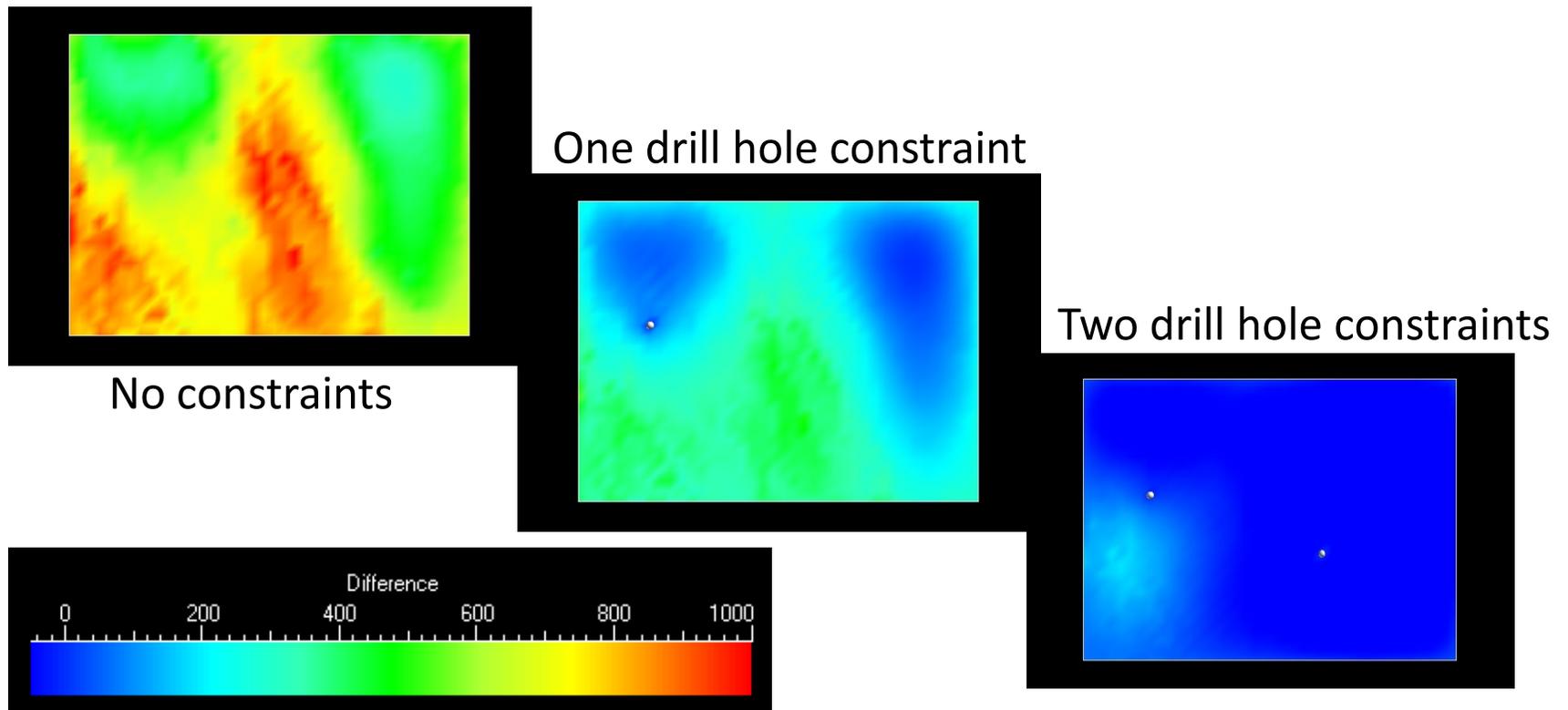
## **Four examples:**

- 2 constraints, huge impact
- 9 constraints, big impact
- Hundreds of constraints, relatively modest impact
- Integrated approach delivers utterly different interpretation



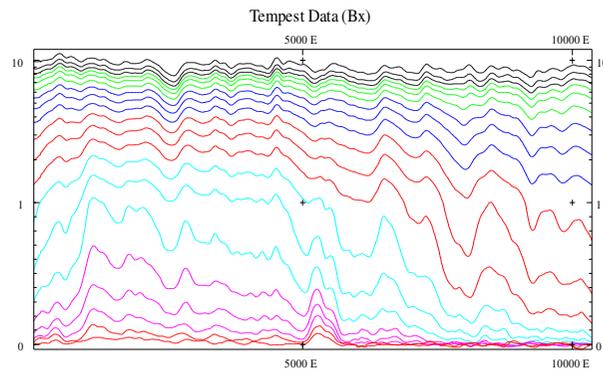
# Constrained geometry inversion of TMI

- Elevation differences between inverted and original basement surfaces

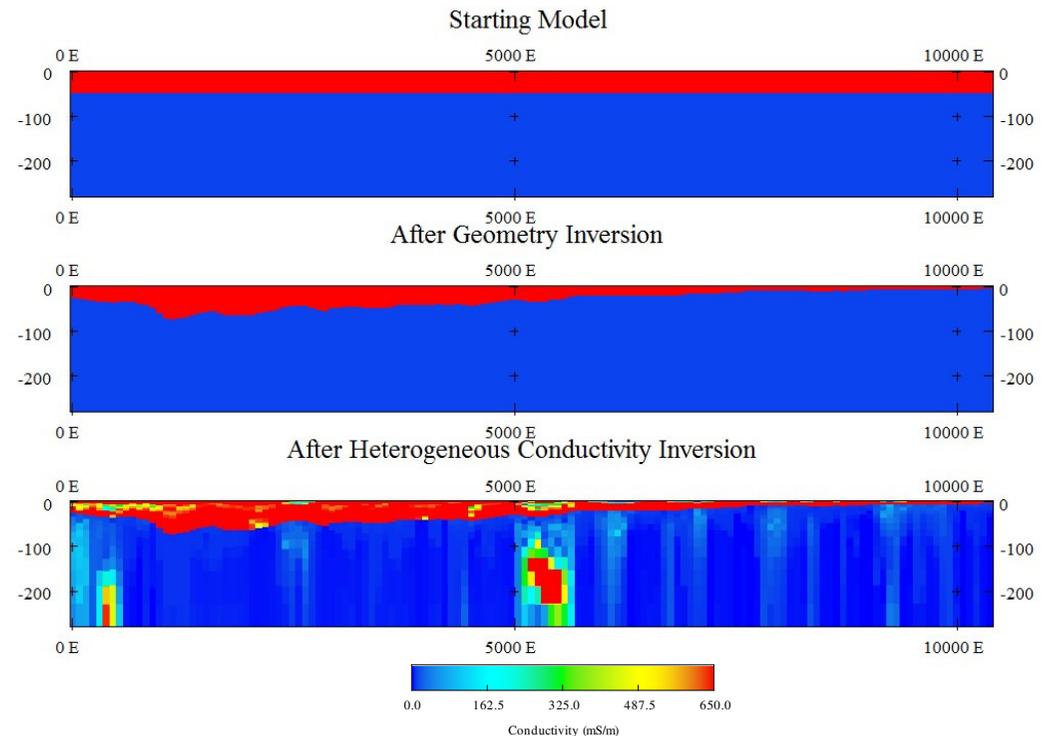


# Tempest survey line – Bull Creek

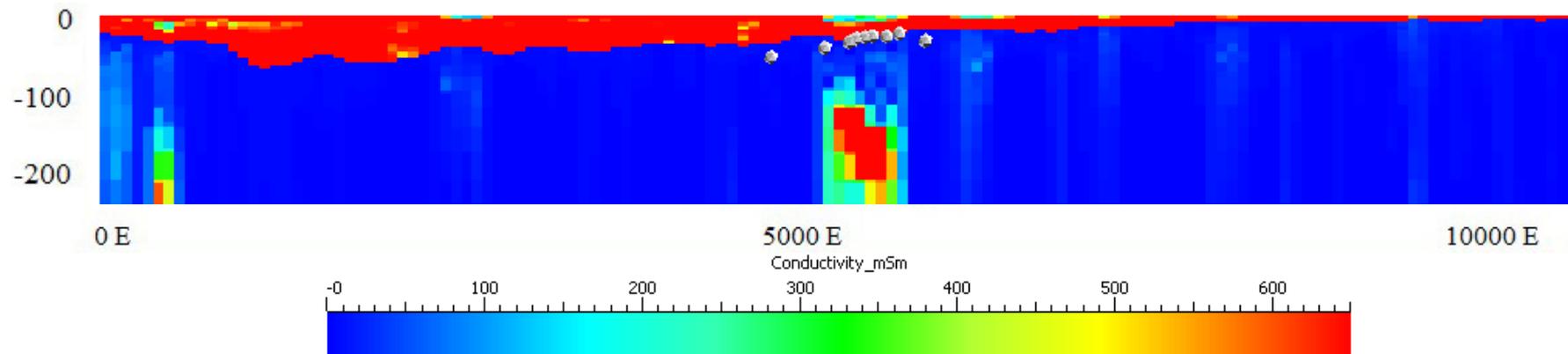
- The starting model comprised cover (50m thick) over basement.
- Cover conductivity was initially 800mS/m (inferred from VPem1D homogeneous conductivity inversion); basement starting value 10mS/m.
- Procedure was depth-to-basement (geometry) inversion followed by heterogeneous conductivity inversion.
- Pierce points constraints not enforced at first.



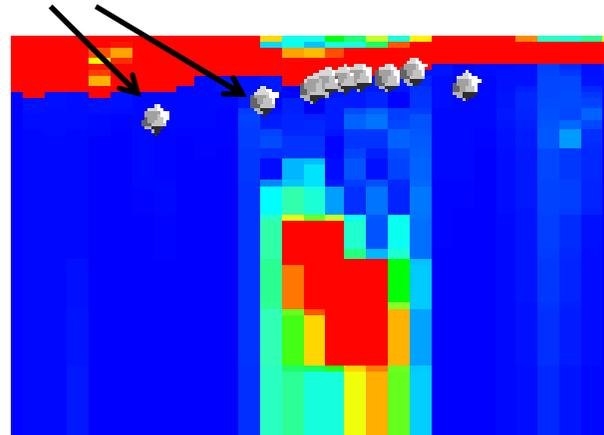
- *Traverse flown over a discrete pyrrhotite rich zone.*
- *Cover thickness varies along the traverse.*



Assess fit to drill hole pierce points after “unconstrained” geometry inversion.

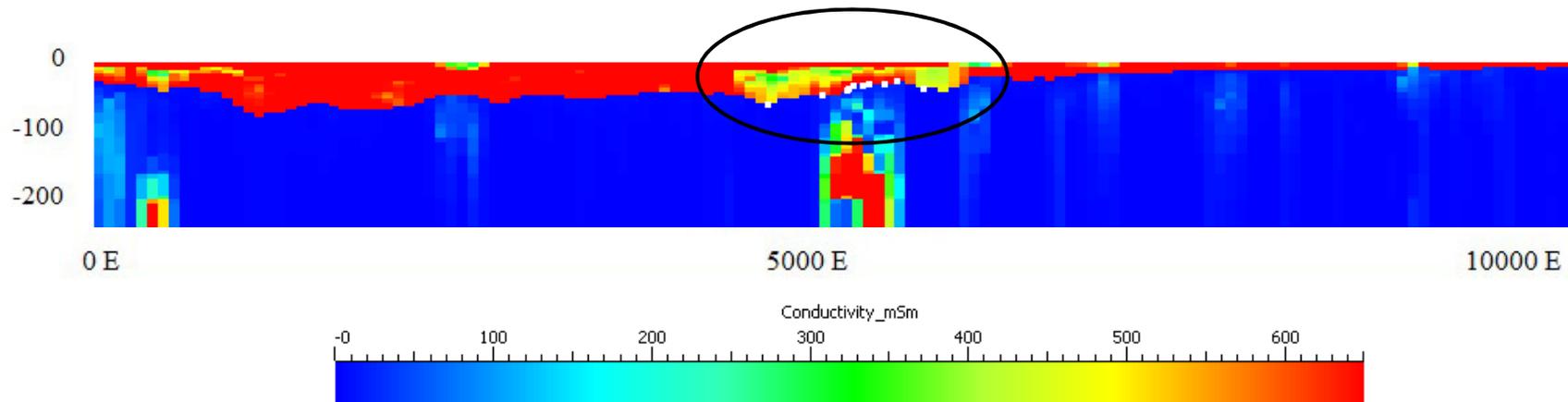


pierce points



# Drill hole constrained inversion – Take 1

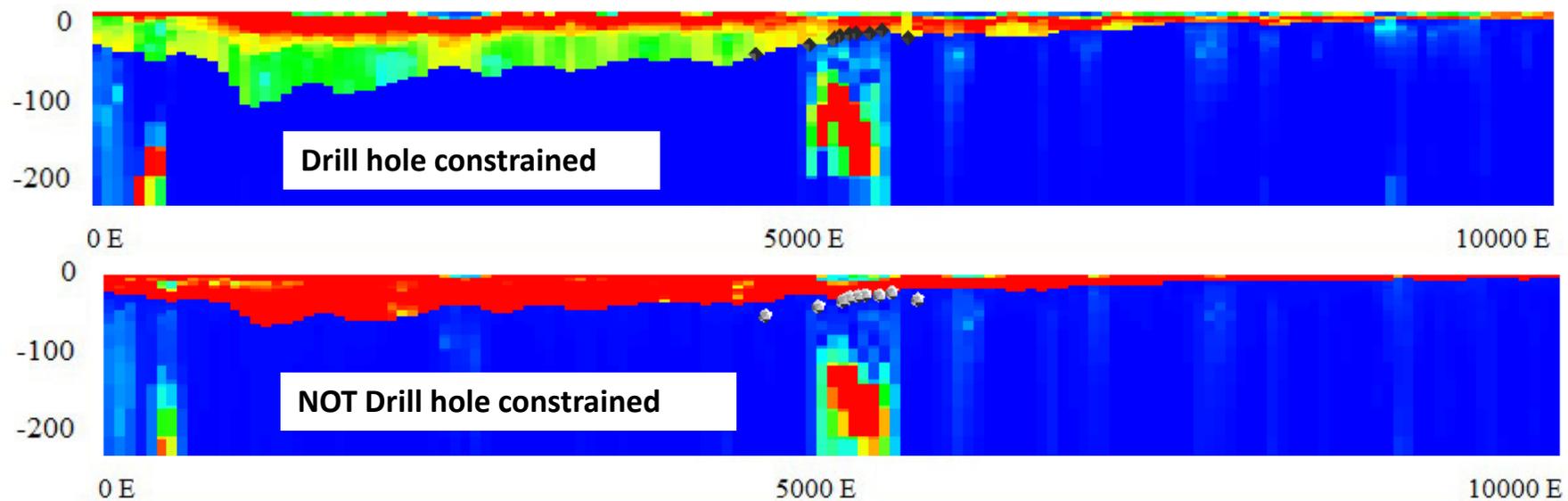
- Re-submit to conductivity inversion with contact adjusted locally to honour drill hole pierce points.



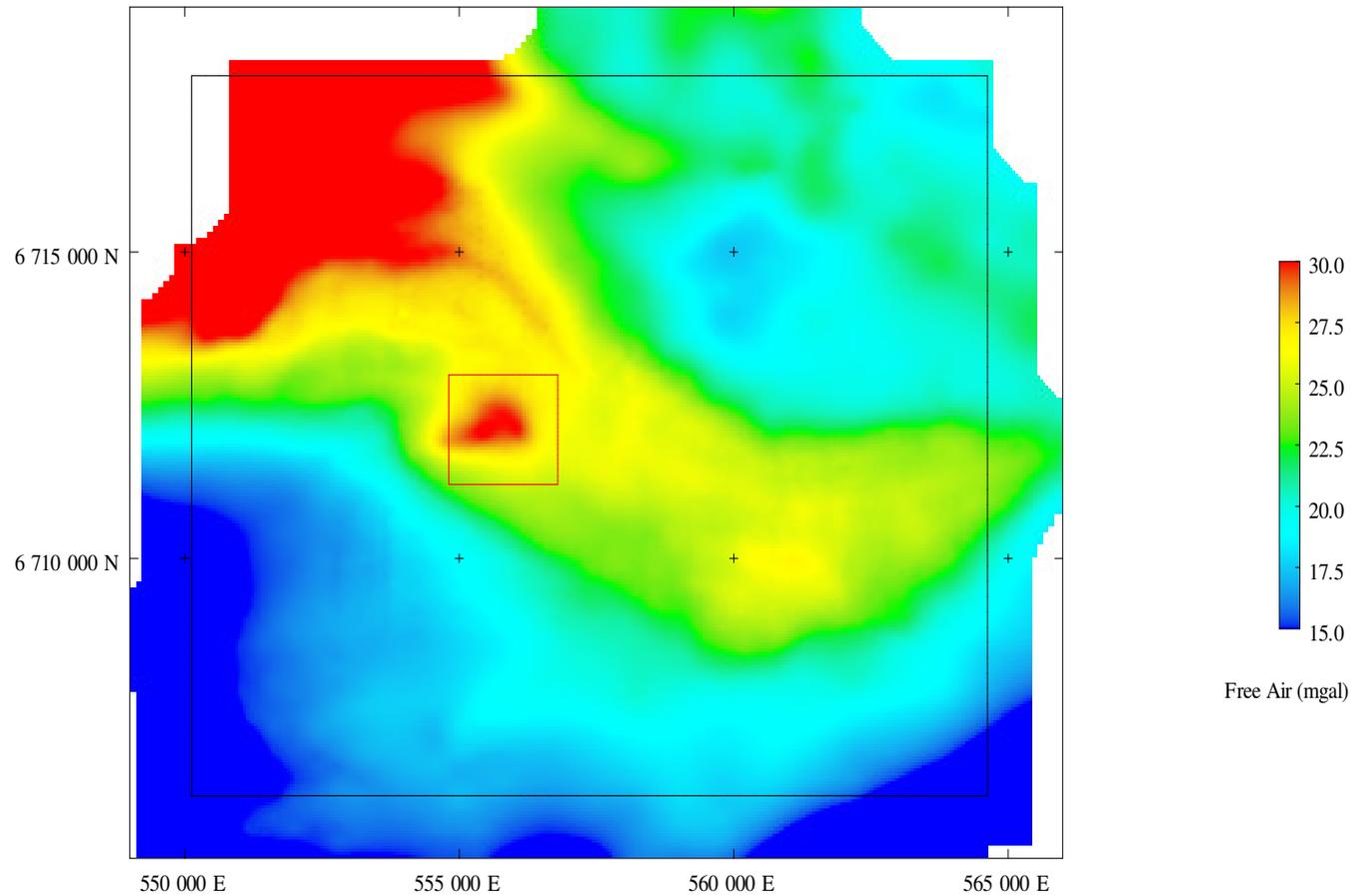
- Cover conductivity in the vicinity of drill holes now appears anomalous.
- Conclude that the assumed starting conductivity for cover may be too high → depth to basement under-estimated.
- Re-run geometry + conductivity inversion sequence, with lower starting model conductivity in the cover (550mS/m c.f. 800mS/m), and honouring pierce points.

# Drill hole constrained inversion

- Updated model consistent with drilling - more geologically plausible
- Drill hole depth-to-basement constraints provided additional control on the starting model conductivity.
- Drill hole constrained inversion suggests multiple layers within thicker cover

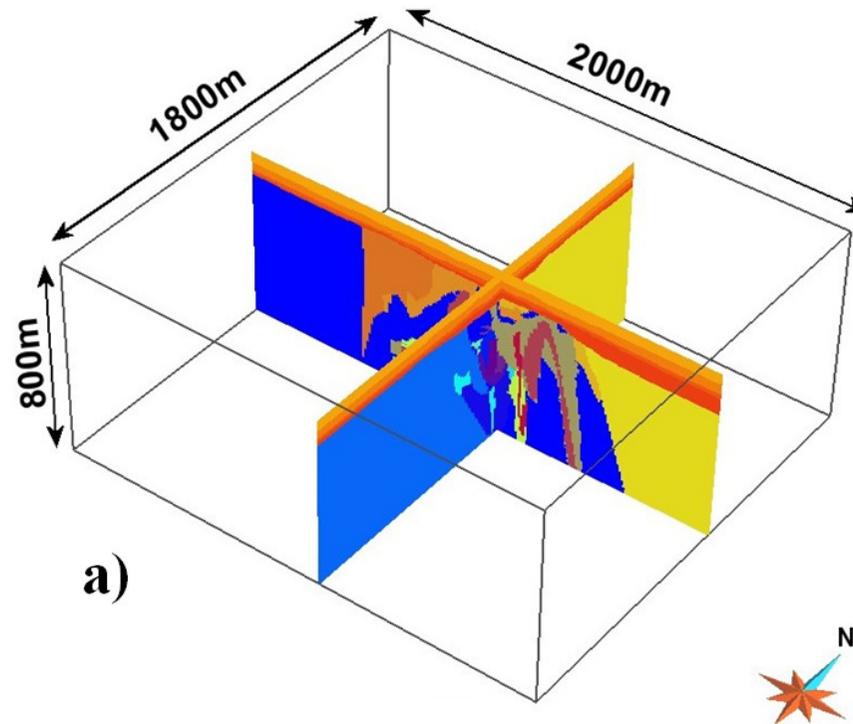
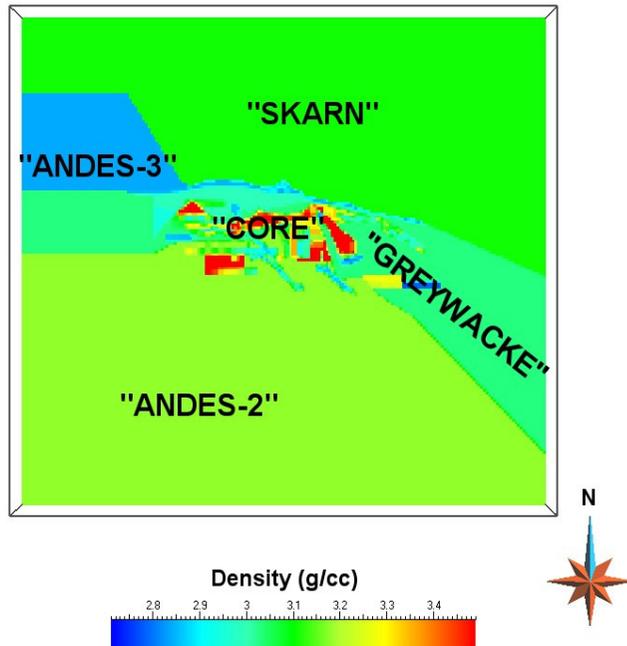


## Prominent Hill (South Australia) regional gravity



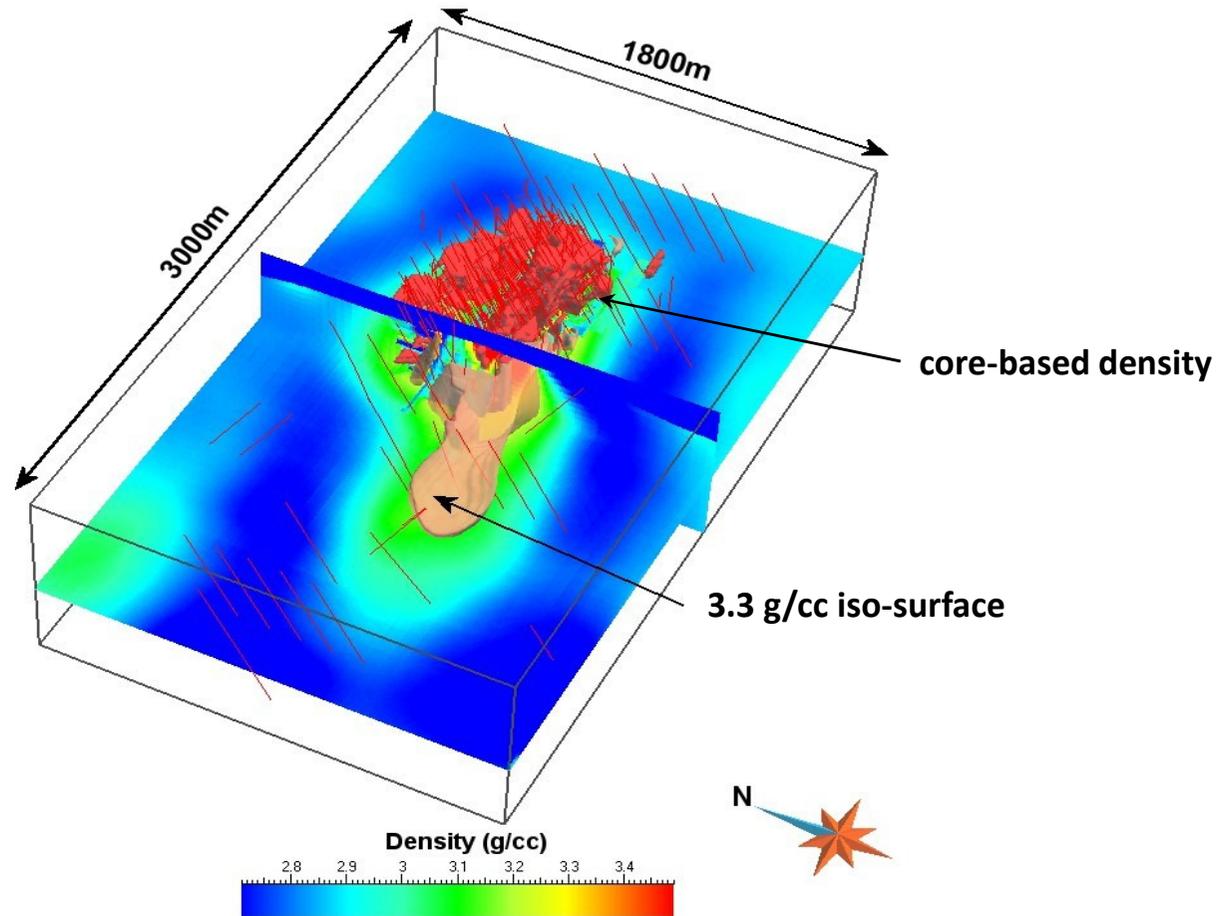
Gridded image of the free air gravity data. The geological model extents are shown in red.

## Prominent Hill geological/density model



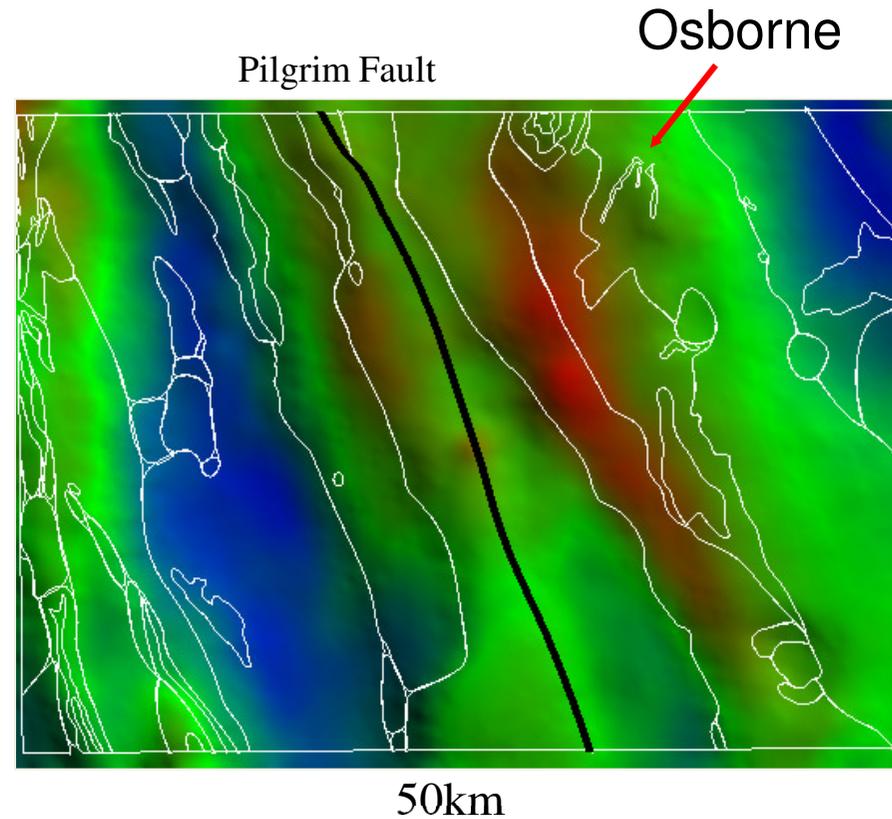
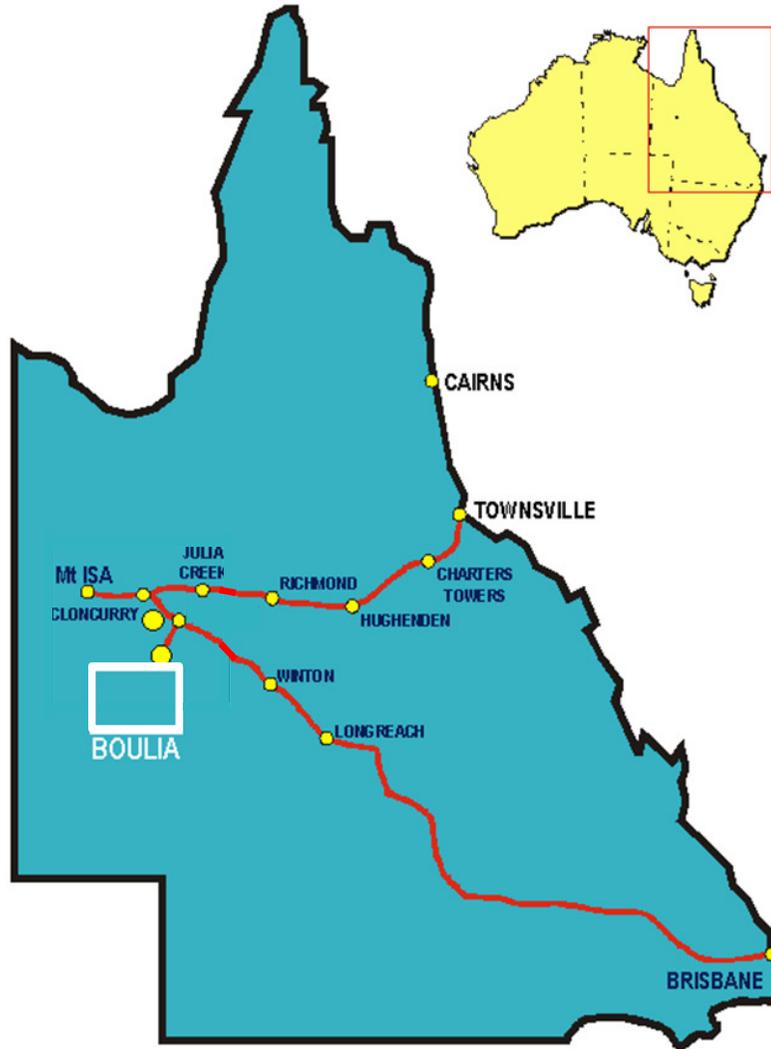
Sections through the supplied density model. Model is very crude outside the 'core' zone.

# Prominent Hill (western extension) density model



Perspective view of the density model after constrained gravity inversion. **Shallow high density tongue** accounts for the western end of the Prominent Hill gravity anomaly. Drill hole trajectories shown in red.

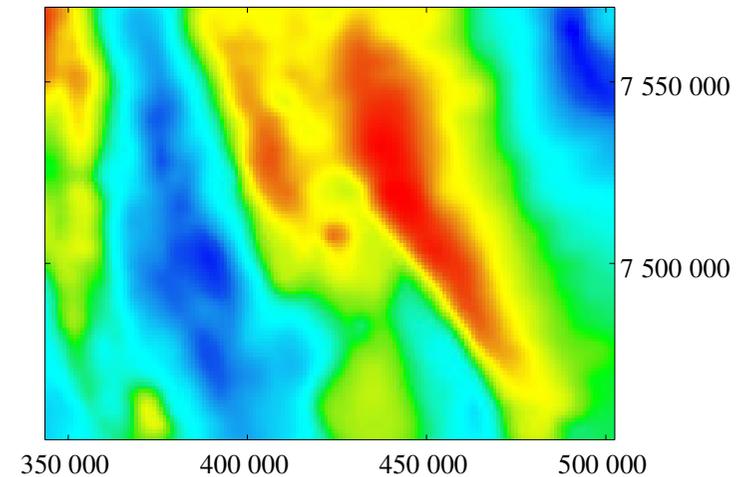
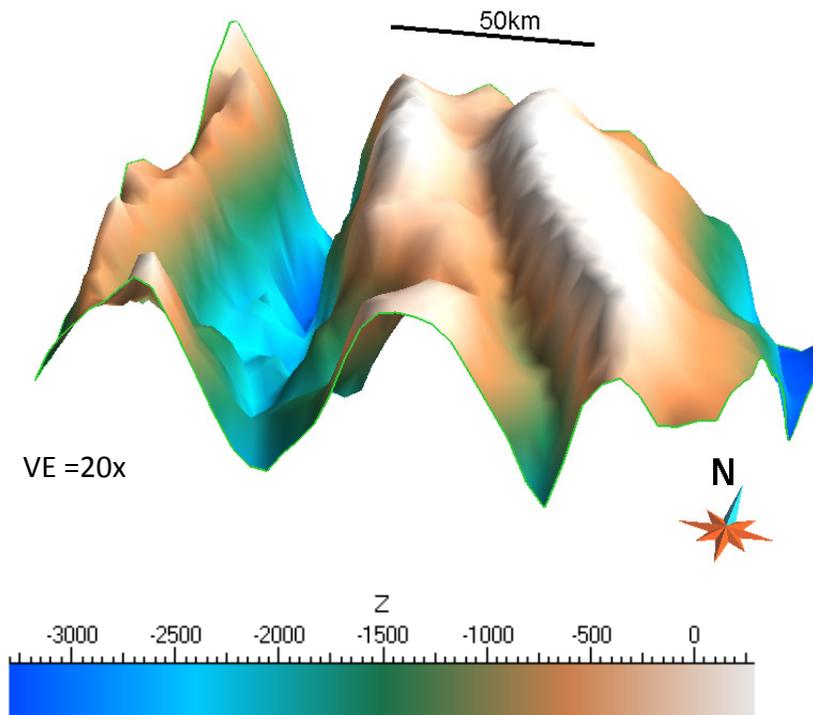
# Geologically- and petrophysically-constrained inversion of Boulia gravity, Queensland



Free Air Gravity Image  
Illuminated from the NE

# “Unconstrained” geometry inversion

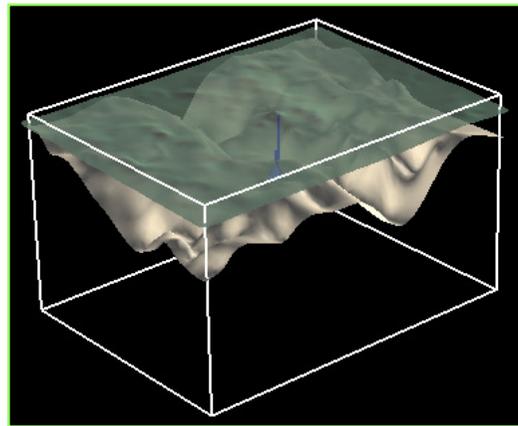
- homogeneous cover (2.4 g/cc) overlying homogeneous basement (2.8 g/cc)
- basement contact horizontal initially at ~700m depth



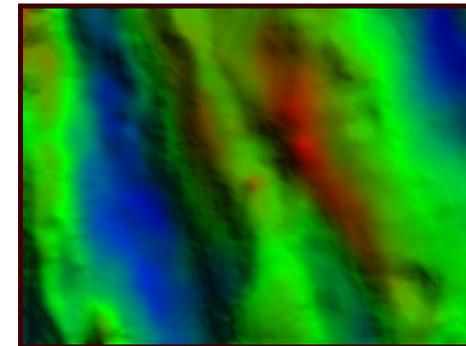
Observed free air gravity data

# Bouli methodology

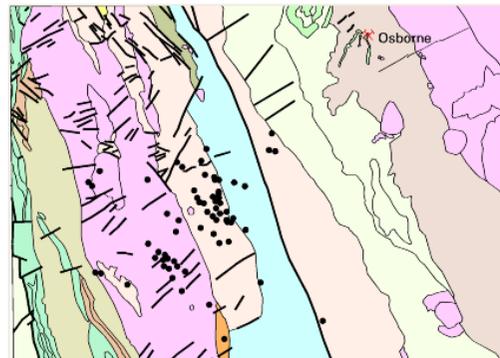
- Compilation of available data



Literature Search  
(Densities)

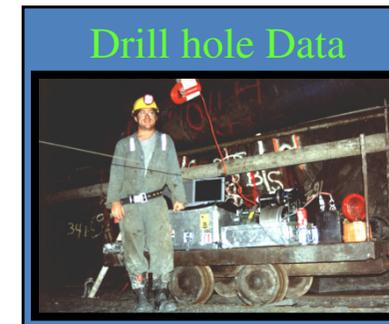


Gravity Data



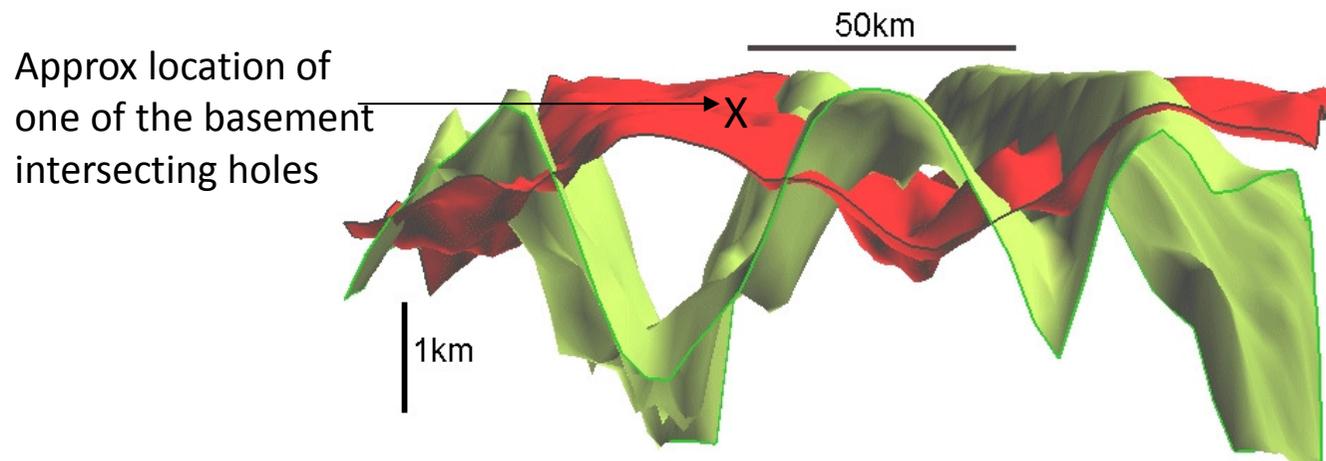
Geology after Mackey et al (2000)

Common Earth Model



# Compare constrained and unconstrained results

- “Unconstrained” and constrained results are markedly different
- Gravity high = basement low after constraints applied



**Red** = basement surface after **constrained** inversion

**Green** = basement surface after **unconstrained** inversion

## Part 3

# Opportunities & roadblocks

# Opportunities

- faster processors in parallel deliver shorter run times
- massive amounts of memory facilitate inversion of larger/more detailed models
- lithological inversion
- efficient joint inversion by exploiting common geology
- incorporation of constraints from seismic
- contribute to resource modelling
- inversion software integrated with geological modelling packages
- inversion by geologists?

# Roadblocks

- inversion still a black box?
- geological model construction and manipulation
- lack of petrophysical data
- time allocated for interpretation
- consultants and contractors at arms length?
- model appraisal and risk assessment
- upscaling
- workflows

# Conclusions

## 1. Need for constraints

- Immediate aim of inversion is to achieve an acceptable data fit, i.e. consistent with error
- Usually an infinite number models which are acceptable
- Need constraints to honour what is known and to reject conflicting models

## 2. Constrained inversion as a component of geological modelling:

- Inversion is a tool, to aid development/refinement of a geological model
- Advantages of inverting on a geological model:
  - Flexibility/control in inversion, e.g. geometry inversion
  - Driver for integration: constraints imposed naturally
  - Capture geological domain attributes, e.g. remanence, stat. distributions

# Conclusions cont'd

## 3. Types of constraints

- Hard/soft; real/artificial
- Wide range of geological constraint options, on geometry, structure, stratigraphy
- Options currently more limited for petrophysical constraints, but potential for lithology prediction and resource characterisation

*Fullagar*  
*Geophysics*  *Pty Ltd*

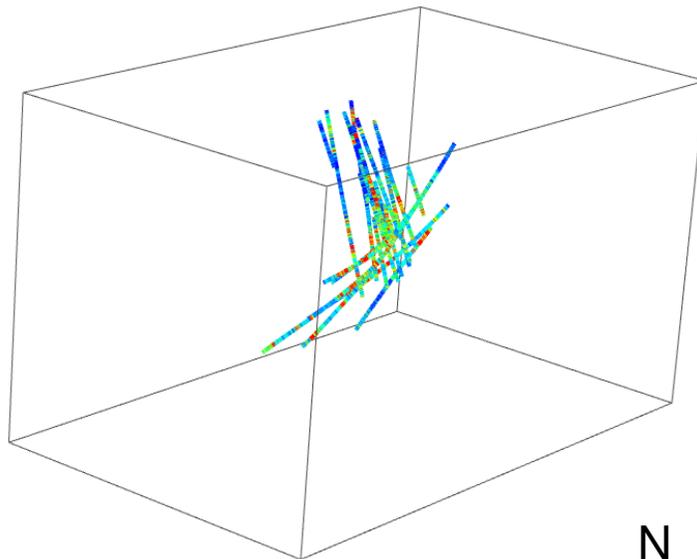
  
**Mira Geoscience**  
*...modelling the earth*

# Model appraisal

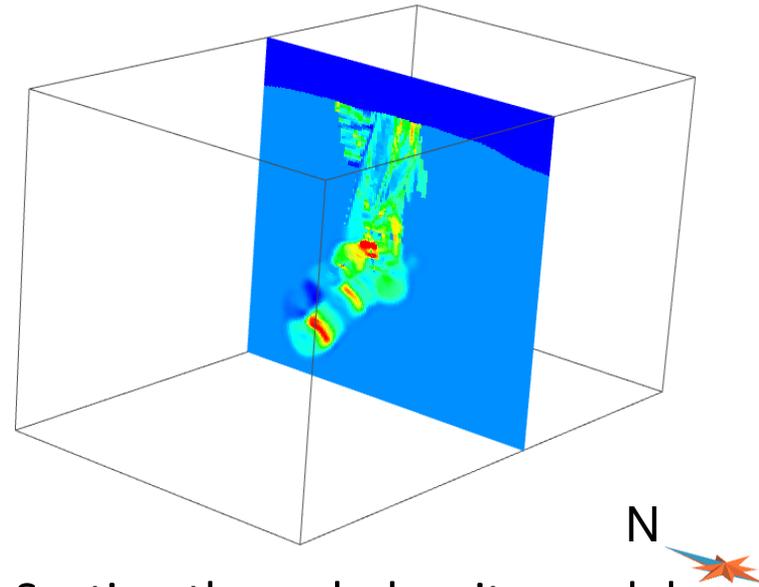
- Stochastic/probabilistic
- “What if?” experiments
- Extremal inversion
- Backus-Gilbert averaging
- Sensitivity analysis

# Prominent Hill (main zone) density modelling

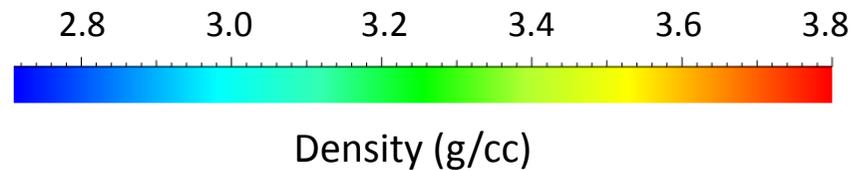
Forward modelling of gravity revealed a mass deficit



Selected drill holes,  
coloured by core density

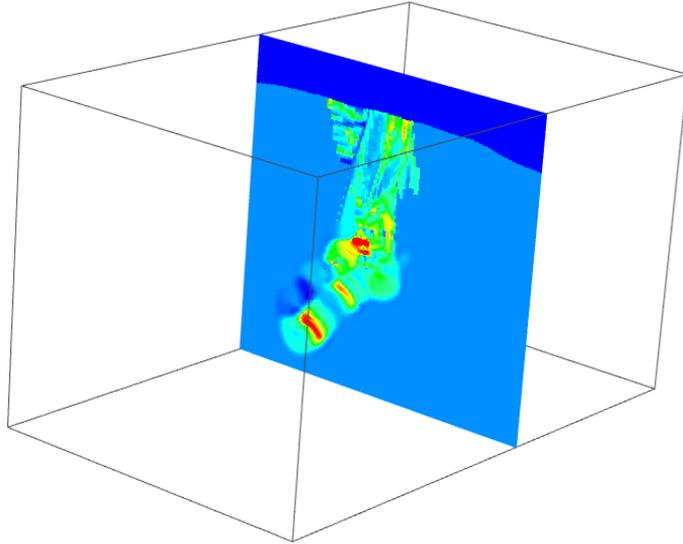


Section through density model  
based on core measurements

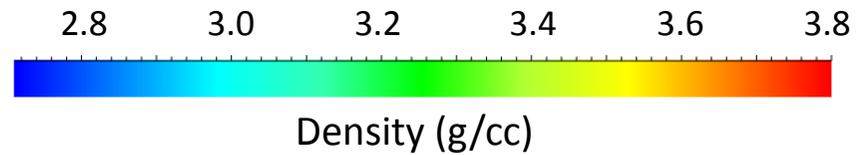
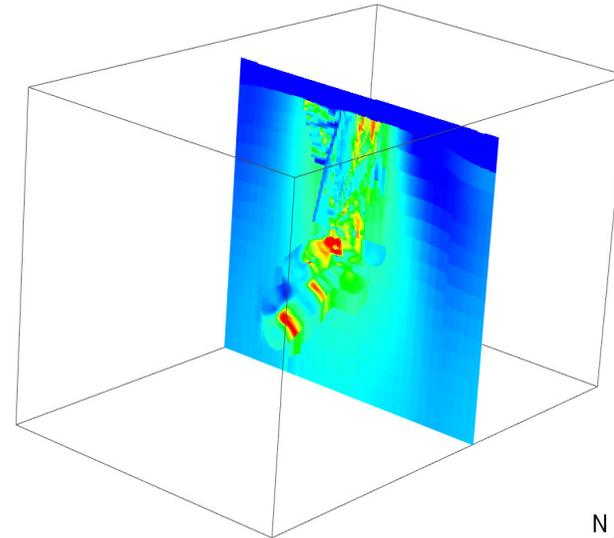


## Prominent Hill constrained gravity inversion

density model  
based on core measurements

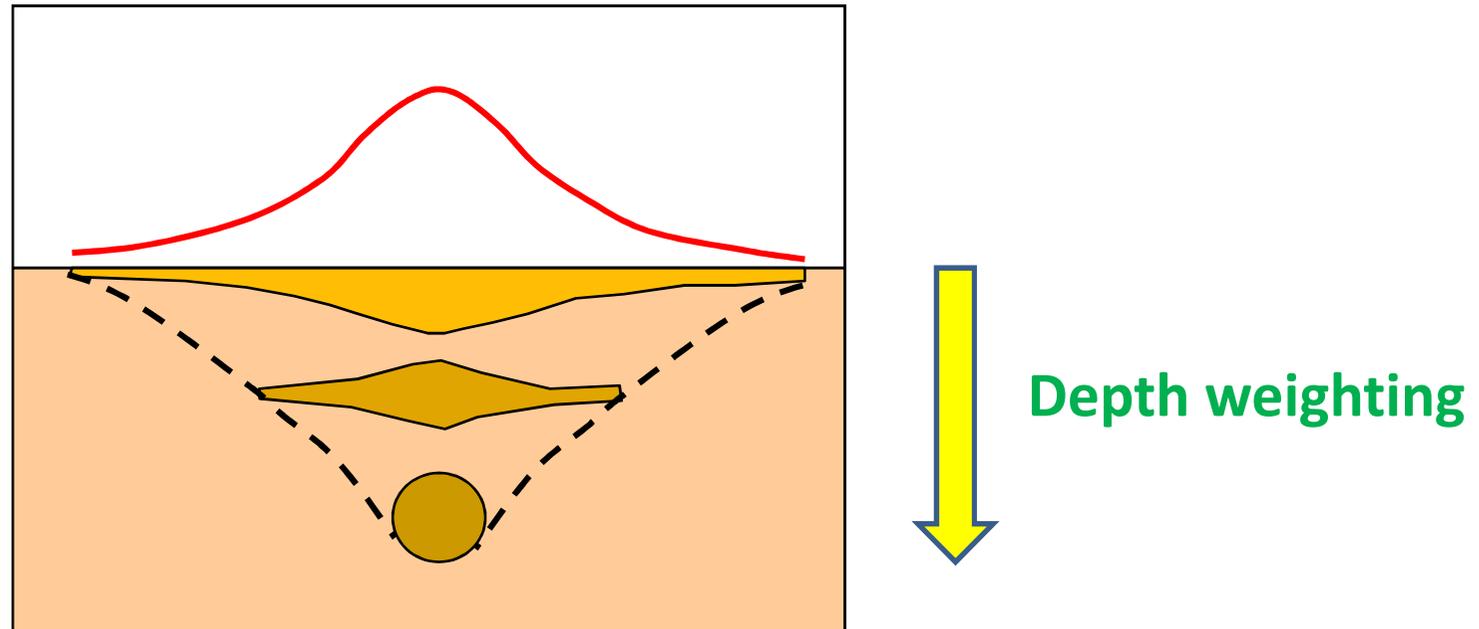


density model  
after constrained gravity inversion.



Inversion defines a **diffuse halo** around the drilled/modelled volume

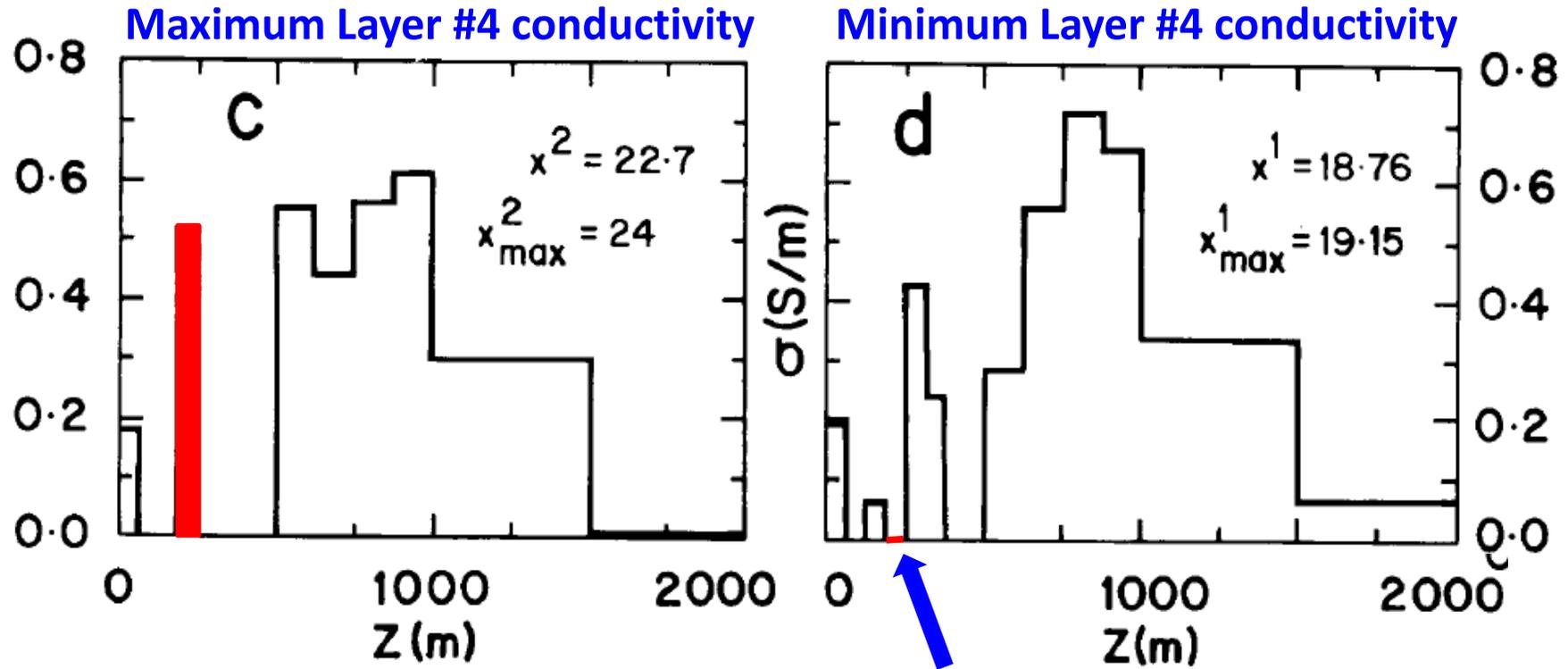
## Extensive & shallow lower density source or compact & deep higher density source?



Re-running the Prominent Hill gravity inversion with depth weighting revealed that the mass deficit *could* be explained by a deep root.

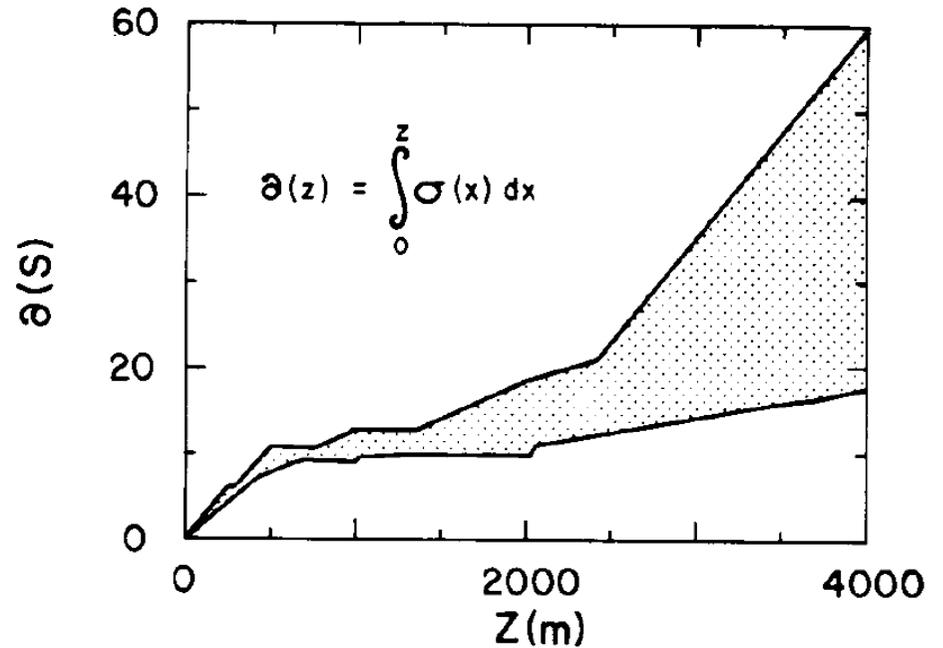
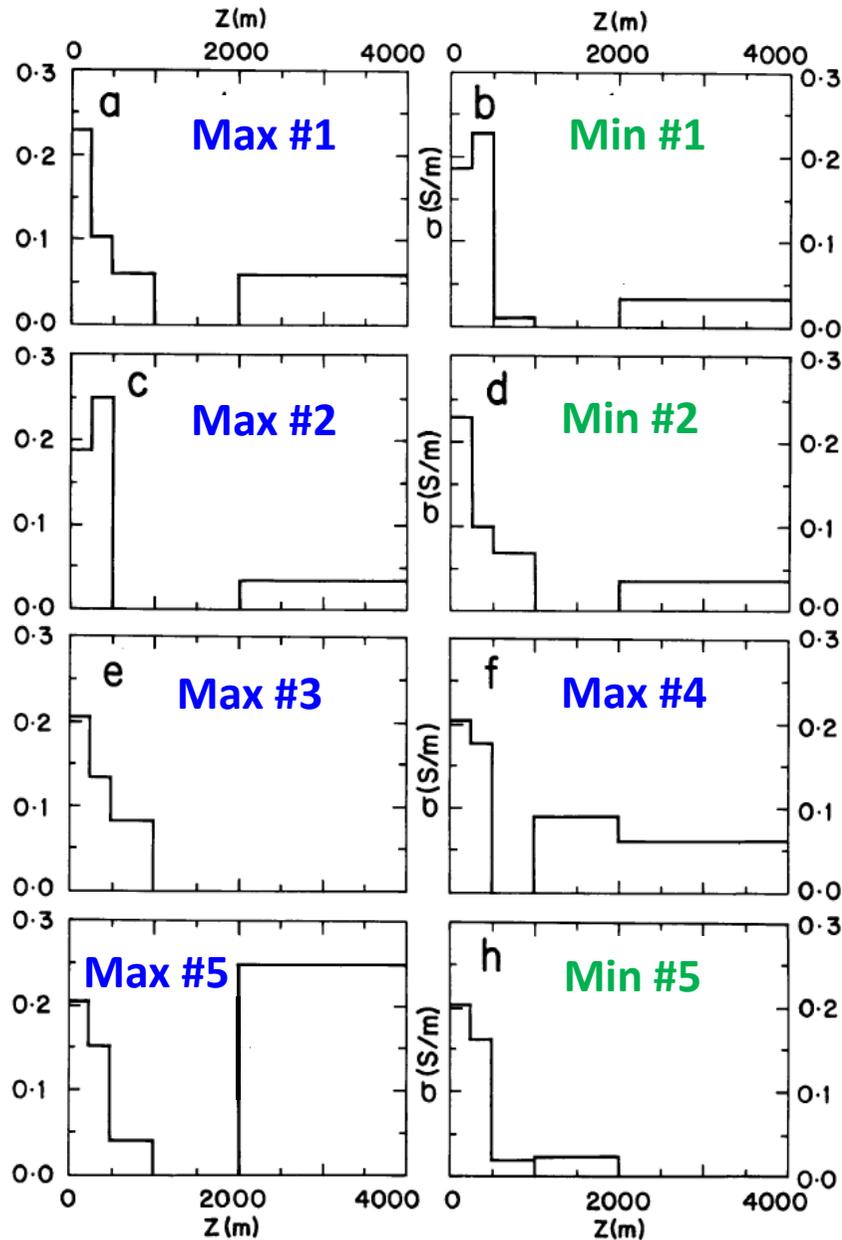
# Appraisal of non-uniqueness via extremal inversion

1D horizontal loop FEM inversion



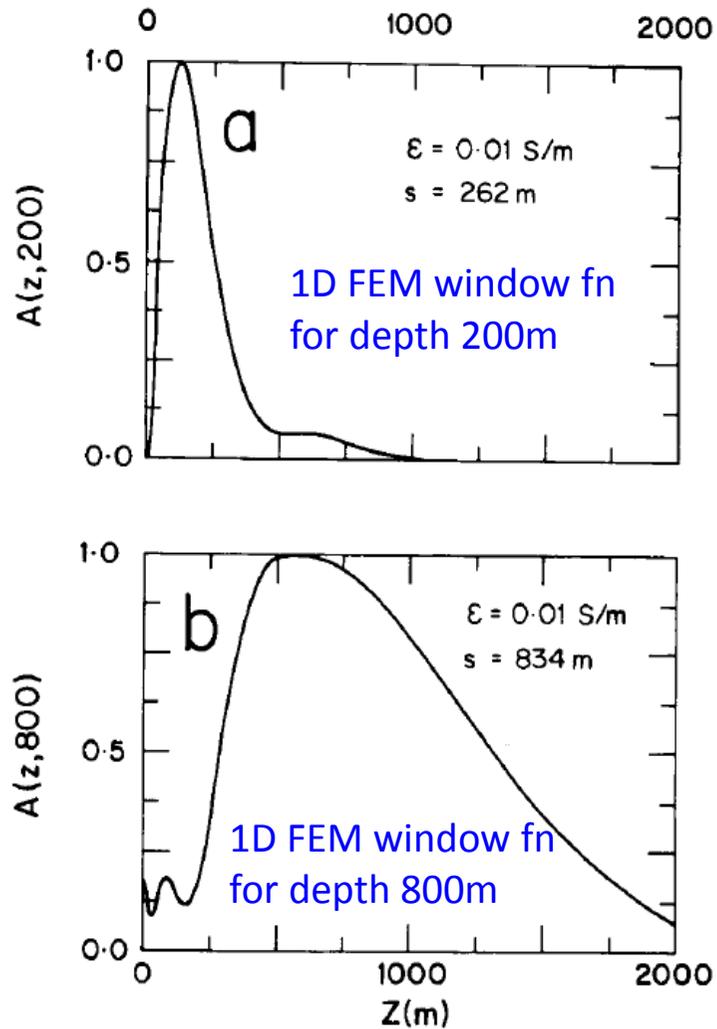
Valid characterisation if layers are geologically meaningful

# Conductance envelope after 1D extremal inversion of FEM



Inensitive to number of layers

# Backus-Gilbert averaging



Any realisable data set has finite resolution

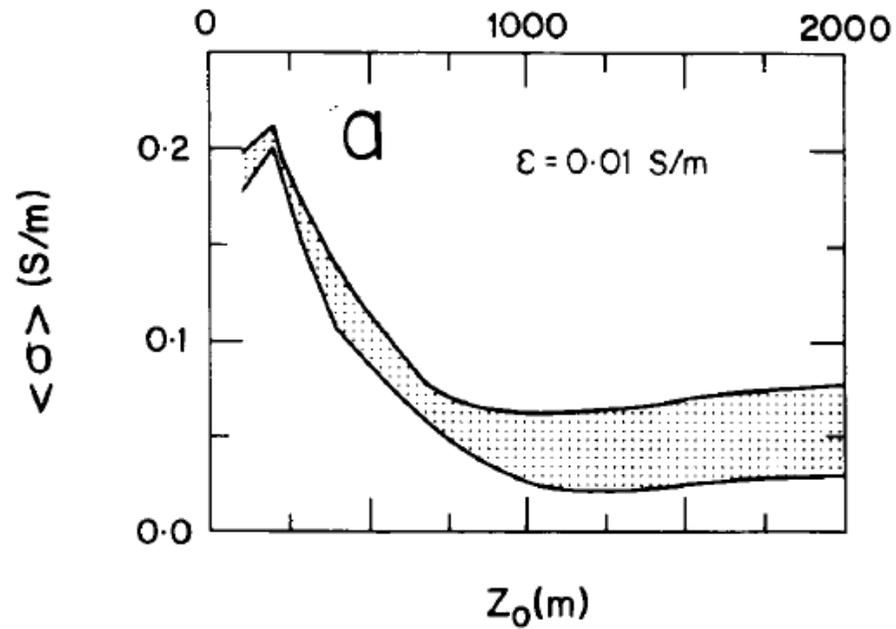
Create “window functions” for depths of interest

Compute averages from acceptable model(s)

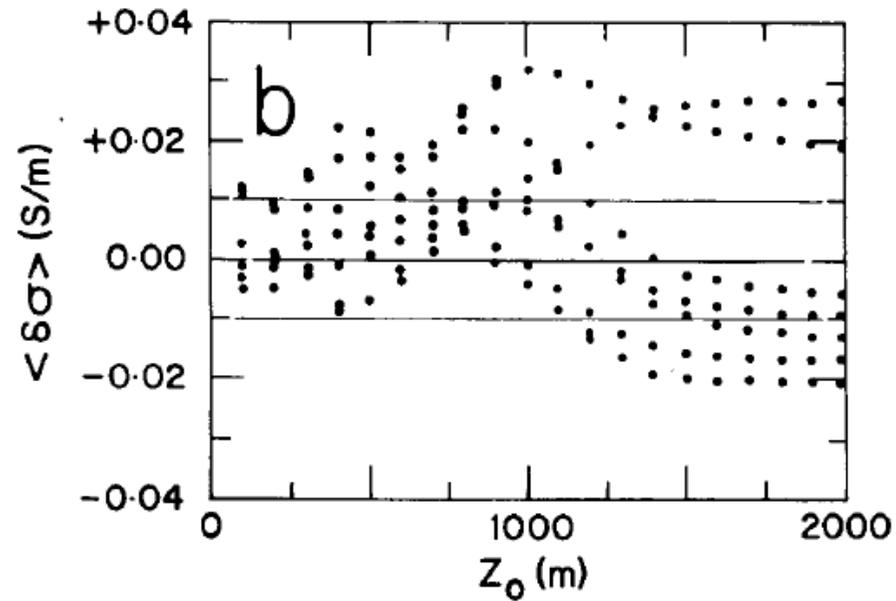
Insensitive to number of layers

Trade-off between resolution and accuracy

# Backus-Gilbert conductivity averages after 1D FEM inversion



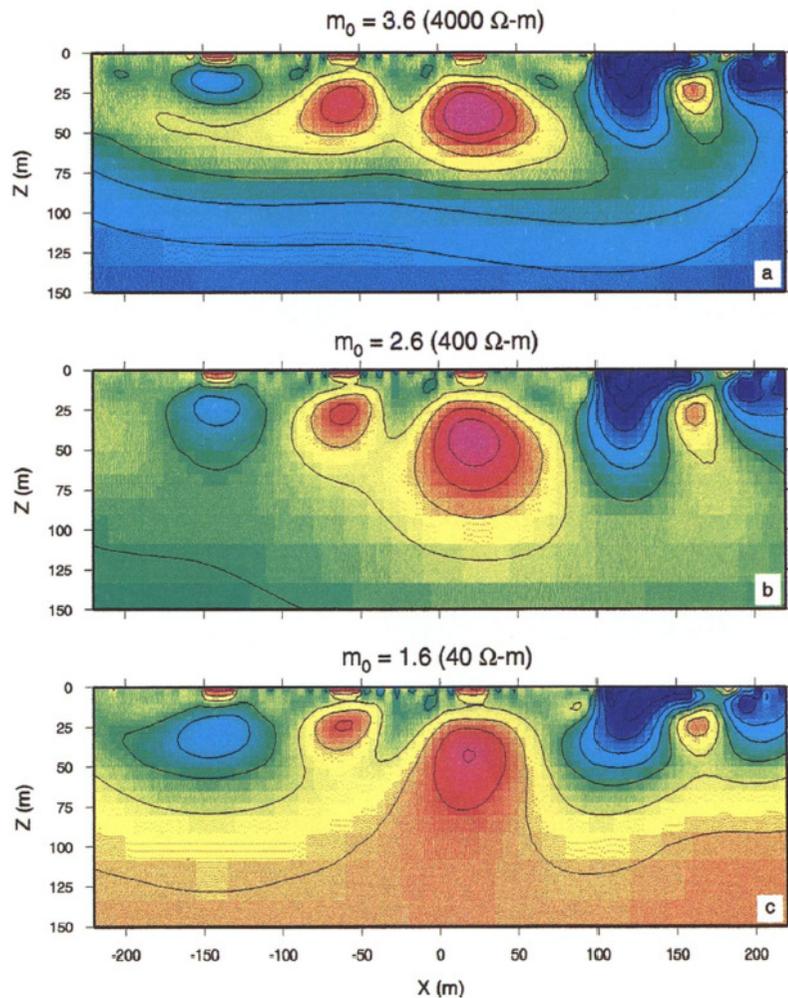
Averages are common to all “linearly close” models



Deviation of extremal model averages from reference model averages

(Fullagar & Oldenburg, 1984)

# Sensitivity analysis



$m_0$  is the starting model resistivity

An important source of non-uniqueness is loss of sensitivity with depth

The depth of investigation (DOI) of a geophysical survey can be defined via successive inversions with different starting models

Features common to all inversions can be considered to be reliable

(Oldenburg & Li, 1999)

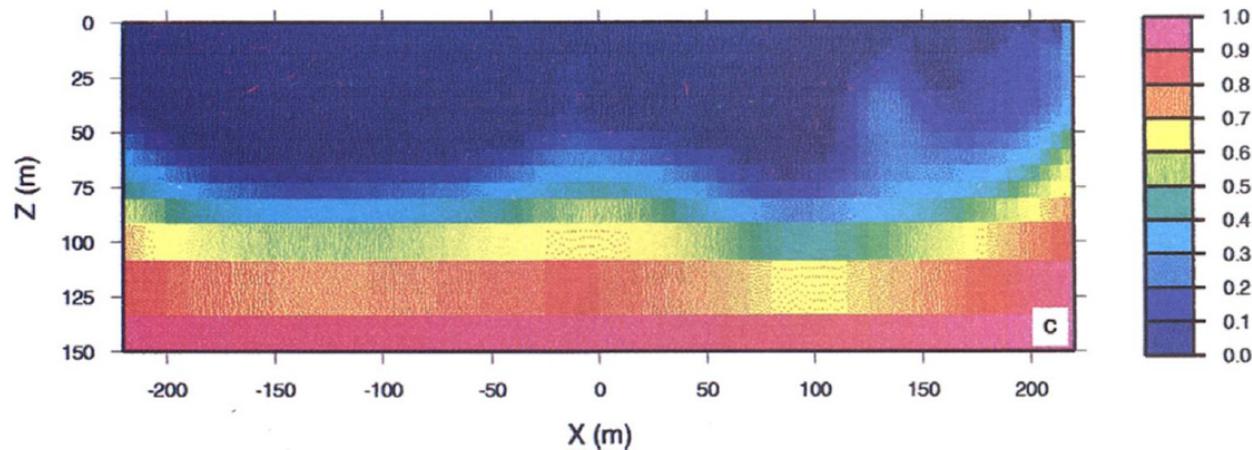
# Depth of investigation (DOI)

(Oldenburg & Li, 1999)

Consider two inversions carried out with constant reference models  $m_{1r}, m_{2r}$ . Let  $m_1, m_2$  be the models recovered, and define

$$\text{DOI index} = \frac{m_1(x, z) - m_2(x, z)}{m_{1r} - m_{2r}}. \quad (3)$$

**DOI index** will approach zero at locations where the two inversions produce the same result regardless of the value of the reference model. We assign high credibility to those areas.  $R$  will ap-



DOI index for  
 $M_0 = 40/4000 \Omega\text{m}$