Introduction to Geophysical Modelling and Inversion

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GEOPHYSICAL INVERSION FOR MINERAL EXPLORERS
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Forward modelling vs. inversion

Forward Modelling: Given a model $m$ and predicting data $d$

\[ d = F(m) \]

$F$ is an operator representing the governing equations relating the model and data.
Inversion

Geophysical inversion refers to the mathematical and statistical techniques for recovering information on subsurface physical properties (magnetic susceptibility, density, electrical conductivity etc) from observed geophysical data.
What is inversion?

Inversion: Recording data $d$ and predicting model $m$

$$m = F^{-1}(d)$$
What is inversion?

Forward Modelling: Given a model and predicting data

$$d = F(m)$$

Inversion: Recording data and predicting model

$$m = F^{-1}(d)$$

Not Possible - Ill Conditioned
Iterative inversion

Starting model and acquisition parameters

Calculate model response using forward modelling algorithm

Compare observed and model responses, and calculate Objective function

Objective function large

Alter model parameters so as to reduce objective function

Objective function small, or maximum no. of iterations exceeded

Inversion process is complete: Output final model

Each cycle through the inversion process is called an iteration
How do inversions work?

This chart summarizes the requirements for proceeding with inversion of geophysical data.

Each box has important implications for successful inversion.

Ability to do forward modelling calculations is assumed.
Models

Model Types

Single Physical Property Value

- Uniform halfspace
- Buried objects
- 1D (layered) models

Parameterized object (susceptibility, length, depth, orientation)

Physical property varies as a function of depth

Plate in a free-space (vacuum)
Plate in a half space
Plate in a layered model

(after: Inversion for Applied Geophysics)
Models

Model Types

- **2D models**: Model is unchanging perpendicular to profile section
- **2.5D models**: Model objects have limited strike length
- **Concatenated 1D models**
- **3D models**: Physical properties change in all 3 directions. Generalized structure

Geologic unit boundaries adjust location to create 3D shapes and bodies.

(after: Inversion for Applied Geophysics)
Why invert data?

Helps explain complex data sets
  e.g. DCIP, Gravity Gradiometry, AEM, ZTEM, DHEM
Removes topography effects

Explains the data with a model(s) of the earth:
  Provides a quantitative model that can be analysed
  → What is the depth, geometry, volume, physical property of the model features?
  More easily relates to geology - easier for interpretation
  → What geologic features can be determined in the model?

Can QC the data, identify problematic data
  acquisition problems

Helps separate the noise from the signal in the data
  estimates the noise levels
  estimates depth of penetration
Example: Target in presence of geological noise

Data are sometimes difficult to interpret

Shallow anomalies represent chargeable boulders in till

Subtle responses are important

Inversion result is more easily interpretable in terms of geology
Know The Data

In order for modelling to occur, all instrument system and survey acquisition parameters have to be known.

In general, try to do as little as possible to the data to preserve the information.

Obviously erroneous data should be removed prior to inversion.

This includes features/anomalies in the data which are not modelled by the forward modelling algorithm e.g., IP or SPM effects in EM data etc.

RUBBISH IN = RUBBISH OUT
Inverse Modelling

Modelling Styles

- **Parametric** – few unknowns

![Diagram](https://via.placeholder.com/150)

**e.g. TEM decay**

**15 Data**

**dB/dt** vs **time**

1D Conductivity model

7 unknown model parameters

(conductivity of each layer; thickness of upper three layers)
Inverse Modelling

Modelling Styles

- **Parametric** – few unknowns
- **Generalized** – many unknowns

- e.g. TEM decay
  - $\frac{dB}{dt}$ vs. time
  - 15 Data

- 1D Conductivity model
  - 40 unknown model parameters
  - 1D Mesh structure predefined but smaller than expected structure of geology.

→ structure inferred from the resulting model
Inverse Modelling

Modelling Styles

• Lithology based
  – VP suite (Fullagar Geophysics)
  – Geomodeller (Intrepid)

• Physical Property based
  – UBC-GIF codes
  – Geosoft Voxi
  – VP suite
Inverse Modelling

Physical Property Based Modelling

- Physical property values of many individual cells are adjusted.
- General structure is recovered

3D susceptibility model

E.g. Magnetic Data

(low value cells removed)
Inverse Modelling

Physical Property Based Modelling

- Physical property values of many individual cells are adjusted.
- General structure is recovered.

Result is a physical property model containing structure

e.g. Magnetic Data

3D susceptibility model

(3D Mesh structure predefined but smaller than expected structure of geology.

10,000+ unknown model parameters

→ structure inferred from the resulting model

RESULT IS A PHYSICAL PROPERTY MODEL CONTAINING STRUCTURE
Inverse Modelling

Lithology Based Modelling

- Provide physical properties (single value or distribution) for each lithology and adjust the geometry to fit the data.

RESULT IS A GEOLOGICAL MODEL

(courtesy Anglo American)
Inverse Modelling

Which Modelling Style to choose?

- Depends on the geophysical method, the survey design, and the exploration goal. Some examples might be:
  - Is the goal to define the geometry/volume? Measure the physical properties well and choose a lithologic based inversion (e.g. VPmg)
  - Is the goal to define a thickness of cover from a few TEM soundings? Use a parametric inversion
  - Is the goal to define both physical properties and geometry? Use a generalized inversion (e.g. UBC)
  - What geologic information is available that can be integrated into the modelling?
Acceptable models and non-uniqueness

There are **infinitely many** models that can explain the observed data.

Why is this so?

- Because there are usually more unknowns (model parameters) than observed data points (underdetermined problem)

- Some physically-based non-uniqueness

- Real data contain noise
Acceptable models and non-uniqueness

There are **infinitely many** models that can explain the observed data.
How to chose one of infinitely many solutions?

Narrow down the range of options using prior knowledge

Geophysical prior knowledge:
- Values are positive, and/or within bounds
- Physical Properties: Estimates for host rock properties
- Point-location values from drill hole information

Logical prior knowledge:
- Find a “simple” result - as featureless as possible.
- This sacrifices resolution but prevents over-interpreting the data.

Geological prior knowledge:
- Character of the model (smooth, discontinuous)
- Some idea of scale length (or size) of the bodies
- Structural Constraints
Model norm

The model norm is a measure of the (mathematical) “size” of a model.

The inversion process is an automated decision making scheme.

The model norm is a way of encoding prior information in a form suitable for mathematical optimisation – we seek the “smallest” model.

The model norm is part of the solution to nonuniqueness …

Nonuniqueness is addressed by choosing the one model (from infinitely many) that minimizes the defined model norm.
Model norms

- Smooth
- Minimum horizontal structure
- Minimum vertical structure

Minimise difference between the model and some “reference model”
Data misfit

Field/ Observed Data → Measure of DIFFERENCE → Predicted/ Calculated Data

Earth

Model
What is a good measure of misfit?

If we assume errors follow a particular distribution then a measure of total misfit between predictions and field data can be defined $\phi_d(m)$:

- Predictions can be considered OK when $\phi_d(m) < \text{tolerance}$
- We don’t want to fit the data too closely or we are fitting noise
- Not all measures of Data Misfit are equal
What contributes to data noise?

Natural and cultural noise sources

Accuracy and precision in data measurements

Data positioning errors

Approximations made in forward modelling
  1D
  2D
  3D
  Plates
  Anisotropy
  Discretization of topography
Measures of misfit

Consider the problem of fitting a straight line to the data shown below:

\[ y = 0.0285x + 0.3333 \]

The residuals are the differences between the data points and the best-fit line at each depth.

They may be positive or negative.
Measures of misfit – L1 and L2 norms
Measures of misfit – L1 and L2 norms

Original data

Misfit = sum of squares of residuals (L2 norm = least-squares)

\[ y = 0.0285x + 0.333 \]
Measures of misfit – L1 and L2 norms

Original data

Misfit = sum of absolute values of residuals (L1 norm)
Measures of misfit – L1 and L2 norms

Add an outlying data point
Measures of misfit – L1 and L2 norms

One outlying data point

Misfit = sum of squares of residuals (L2 norm = least-squares)
Measures of misfit – L1 and L2 norms

One outlying data point

Misfit = sum of absolute values of residuals (L1 norm)
Combining model norms and misfit

A statement of the inverse problem is:

Find the model which

Minimises the model norm ($\phi_M$), and

Produces an acceptably small data misfit ($\phi_D$)

Mathematically, this becomes a single optimisation

“Minimise $\phi = \phi_D + \beta \phi_M$” (subject to $\phi_d < \text{tolerance}$)

$\phi$ is the combined objective function
$\beta$ is the trade off parameter (regularisation parameter)
**β** is the regularization parameter

- **Solve:**
  \[ \phi(m) = \phi_d(m) + \beta \phi_m(m) \]

- **β** too large ⇒ underfitting the data. *Structural information lost.*
- **β** too small ⇒ overfitting the data. *Noise becomes imaged as structure.*
- **β** just right (\( \phi_d \approx N \)) ⇒ optimal fit. *Best estimate of a model which adequately re-creates the observations.*
Inverse Modelling

Non-Uniqueness: Solution (partial…)

- Provide explicit geological information
  - Constraints

- Combine information from independent geophysical methods
  - Joint or Cooperative Inversions
    e.g. Gravity with Magnetics, Airborne EM with CSAMT, etc.
Integrated Modelling: Constraints

Sources of Data

- Geologic Mapping
- DH geological logs
- Interpreted cross-sections
- 3D geological models
- Physical property data per lithology
- Located physical property data measurements

Some information is subjective and some information is objective.

As with the geophysical data we would desire to quantify the uncertainty associated with this data as an input to the inversion.
Rock properties are the link between geology and geophysics.
Shameless plug – Mira Geoscience Rock Property Database System

http://rpds.mirageoscience.com/

6 million measurements, including GSC database and published data
Organise, understand, preserve and provide access to physical property data
Common Earth Modelling

Extending the model to include multiple properties, that honour multiple data sets, on a single model object.

**Goal:**
Obtain the most complete representation of the earth.

**Benefits:**
Improved resolution away from constraints

Allows more precise exploration using quantitative 3D GIS analysis.
Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic
Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic

Geophysical data

Default UBC-GIF inversion

Medium-high density
High density
Medium density
Low density

Sampling

Drilling

Mapping

3D models

(Nick Williams)
Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic

Geophysical data

Default UBC-GIF inversion

Medium-high density
High density
Low density

Medium density

Sampling

Drilling

Mapping

3D models

Geologically-constrained UBC-GIF inversion

Regolith & sediment

Sulphides

Ultramafics

Granite?

Ultramafics?

Metamorphics?

(Nick Williams)
Surface constraints can result in dramatic improvements.

Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic

Geophysical data

Default UBC-GIF inversion

Medium-high density
High density
Medium density
Low density

Sampling
Mapping

Drilling

3D models

Geologically-constrained UBC-GIF inversion

Regolith & sediment
Ultramafics?
Metamorphics?
Sulphides
Ultramafics
Granite?

(Nick Williams)
Joint and cooperative inversion

Inversion using more than one geophysical method

Methods sensitive to same physical property (e.g., TEM and CSAMT)

Methods sensitive to related properties (e.g., seismic and gravity)

Joint inversion – single objective function

Cooperative inversion – iterative/sequential approach

These approaches require that we establish relationships between the physical properties each method is sensitive to
Appraisal – How good is our model?

Over-fitting vs under-fitting data
Limits to the data
Limits to the physics
Depth of investigation

Suite of models

Point-spread functions
Model resolution analysis
Sensitivity analysis
Extremal models
Model Covariance Matrix
Co-Kriging error
Summary and conclusion

Inversion has the potential to greatly improve the geological interpretation of geophysical data

- High quality data is essential for the success of geophysical modelling
- More appropriate/efficient surveys can be designed
- Complex data sets can be understood (DH IP, 3D EM)

Understanding physical property data is the key to successful inversion interpretation.

- Rock type
- Alteration
- Mineralization
Summary and conclusion

Non uniqueness in inversion is dealt with by imposing constraints
- Provide the constraints or they will be provided for you
- Minimum structure or geological

Interpretation of inversion requires understanding of which parts of the model are driven by constraints and which parts are driven by data.
- Requires inspection of multiple models

Inspect observed and predicted data before accepting a model.
- Did the inversion fit the data anomalies you are interested in?
- Beware of over-fitting and under-fitting your data
Summary and conclusion

Geologically constrained inversion will greatly improve your results

- Constraints can be factual or conceptual (hypothesis testing)
- Sparse or detailed
- From different sources

Geological maps
Outcrop samples
Estimates of overburden depth
Detailed drill data
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Reference

Inversion for Applied Geophysics
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